GEOMETRIC AND ANALYTIC GROUP THEORY
VENTOTENE, 9-14 SEPTEMBER 2013
ABSTRACTS

Diophantine properties of nilpotent Lie groups

Menny Aka (ETHZ)

Abstract: I will present a joint work with Emmauel Breuillard, Lior Rosenzweig and Nicolas de Saxc. A finitely generated subgroup Γ of a real Lie group G is said to be Diophantine if there is $\beta > 0$ such that non-trivial elements in the word ball $B_\Gamma(n)$ centered at the identity never approach the identity of G closer than $|B_\Gamma(n)|^{-\beta}$. A Lie group G is said to be Diophantine if for every $k > 0$, a random k-tuple in G generates a Diophantine subgroup. Semi-simple Lie groups are conjectured to be Diophantine but very little is proven in this direction. I will discuss a recent characterization of Diophantine nilpotent Lie groups in terms of the ideal of laws of their Lie algebra. Somewhat surprisingly, this enables us to consrtuct examples of non Diophantine nilpotent and solvable (non nilpotent) Lie groups. It also follows from the characterization that nilpotent Lie groups of class at most 5, or derived length at most 2, as well as rational (or algebraic) nilpotent Lie groups are Diophantine. Our preprint is available at http://arxiv.org/abs/1307.1489.

Uniformly finite homology and amenable groups

Matthias Blank (Universität Regensburg)

Abstract: We give a general introduction to uniformly finite homology and homology with ℓ^∞-coefficients and discuss their relation in the case of finitely generated groups. Then, we give an overview about known applications, in particular regarding questions about amenability and rigidity of groups. Finally, we present our calculation of uniformly finite homology of many amenable groups.

This talk is based on joint work with Francesca Diana.

Date: September 4, 2013.

The Chern Conjecture for affine manifolds

Michelle Bucher (Université de Genève)

Abstract: Solving Hilbert's 18th problem, Bieberbach showed that the fundamental group of any Euclidean manifold M is virtually abelian and in particular M has vanishing Euler characteristic. Enlarging the class of manifolds to affine manifolds, the corresponding statements are highly conjectural:

1) The Auslander Conjecture (1964) states that the fundamental group of any complete compact affine manifold is virtually solvable, and is known to hold in small dimensions only.

2) The Chern Conjecture (1955) predicts that any compact affine manifold has vanishing Euler characteristic. It is known for surfaces (Benzecri), complete affine manifolds (Kostant-Sullivan), irreducible locally symmetric spaces of higher rank (Goldman-Hirsch), local products of surfaces (B-Gelander).

In this talk I will give an introduction to affine manifolds and present a result with Tsachik Gelander which proves a stable variant of the Chern Conjecture: For any manifold M, there exists K such that for every $k > K$ the product $M \times S^k$, where S is a higher genus surface, satisfies the Chern Conjecture.

Abelian versus free groups acting on buildings

Corina Ciobotaru (Université de Louvain)

Abstract: By making use of strongly regular hyperbolic automorphisms of Euclidean buildings, we prove that any group G (not necessarily discrete) acting co-compactly by type-preserving automorphisms on a locally finite thick (general) building contains $\mathbb{Z}^d$-by-compact, for some natural number d, and satisfies Tits' alternative.

Random walks, Hodge theory, and rigidity

Simion Filip (University of Chicago)

Abstract: The group $\mathrm{SL}(2, \mathbb{R})$ naturally acts on the space of surfaces with a flat metric and a distinguished direction (i.e. holomorphic 1-forms on a surface). The dynamics of this action has applications to more classical dynamics (e.g. billiards and interval exchanges) but is also interesting in analogy with homogeneous dynamics. The underlying structure is also very rich and related to questions in algebraic geometry. I will begin with a description of what the objects of interest are and some illustrative examples. I will then explain how random walks, combined with Hodge theory, can give rigidity results for the dynamics.

Higher dimensional cost and deficiency-gradient

Damien Gaboriau (ENS Lyon)

Abstract: If Γ is a finitely presented and residually finite group, we define the **deficiency-gradient** along a chain of finite index normal subgroups with trivial intersection as

$$\mathrm{def}-\mathrm{grad}\,(\Gamma;(\Gamma_n)_n) := \lim_{n\to\infty} \frac{\mathrm{def}(\Gamma_n)}{[\Gamma:\Gamma_n]}$$

where $\mathrm{def}(G)$ is the deficiency of G.

This is an analogue of the rank-gradient introduced by M. Lackenby.

The goal of my talk is to explain how we compute the deficiency gradient along any chain for such groups as

$$\begin{array}{ll} \Gamma = \mathrm{MCG}(\Sigma_{g,p}), \quad g>2 & \mathrm{def}-\mathrm{grad}\,(\Gamma;(\Gamma_n)_n) = 0 \\ \Gamma = \mathrm{SL}(d,\mathbb{Z}), \quad d>3 & \mathrm{def}-\mathrm{grad}\,(\Gamma;(\Gamma_n)_n) = 0 \\ \Gamma \text{ a limit groups} & \mathrm{def}-\mathrm{grad}\,(\Gamma;(\Gamma_n)_n) = \beta_1(\Gamma) \end{array}$$

where β_1 is the first ℓ^2-Betti number.

Indeed, we identify the deficiency gradient as a **higher dimensional** 2**-cost** defined as the optimum deficiency of “measured leaf-simply-connected laminations” spanning the “action of Γ on the projectiv limit of the equiprobability preserving multiplication actions $\Gamma \curvearrowright \Gamma/\Gamma_n$”.

This is joint work with M. Abert.

Random walks, boundary values and vanishing of ℓ^p-cohomology

Anthoine Gournay (Université de Neuchâtel)

Abstract: Given a graph (of bounded valency), there is a natural operation from function on vertices to function on edges given by the gradient (the difference at the extremities of the edge). Simply put, the ℓ^p-cohomology (in degree 1) of this graph is the quotient of the space of functions with gradient in $\ell^p(E)$ by the space of functions in $\ell^p(X)$. When the graph is the Cayley graph of a group G, this coincides with the cohomology of the left-regular representation of G in $\ell^p(G)$. These quotient are also useful invariants (of quasi-isometry).

In this talk, a natural map from reduced ℓ^p-cohomology to harmonic functions (under relatively mild growth assumptions on the graph) will be constructed. Among the consequences, a partial answer to a question of Pansu for the will be given, namely, for the Cayley graph of a group of superpolynomial growth: if there are no non-constant bounded harmonic functions whose gradient is ℓ^p, then the reduced ℓ^q-cohomolgy is trivial for all $q < p$.

This allows to make significant progress on a question of Gromov (whether the reduced ℓ^p cohomology for all amenable groups and all $1 < p < \infty$ vanishes or not). First, one gets that the reduced ℓ^p-cohomology is trivial for all $1 < p < 2$ and all amenable groups. Second, if the group is Liouville, then the reduced ℓ^p-cohomology is trivial for all $1 < p < \infty$.

ON INVARIANT RANDOM SUBGROUPS OF LINEAR GROUPS

Yair Glasner (Ben Gurion University of the Negev)

Abstract: Let G be a countable linear group with simple Zariski closure. We prove that there exists a free subgroup $F < G$ such that for every non-free action of G on a probability space (X, B, μ) by measure preserving transformations the following hold (almost surely)

(1) F and G have the same orbits.
(2) Two stabilizers $Fx = Fy$ are equal if and only if $Gx = Gy$.

ON RANDOM GROUPS OF INTERMEDIATE GROWTH

Slava Grigorchuck (Texas A&M University)

Abstract: In 1968 it became apparent that all known classes of groups have either polynomial or exponential growth and John Milnor formally asked whether groups of intermediate growth exist. In 1984, I introduced the first such examples and in fact constructed a continuum of groups with different intermediate growth, so, in particular, these groups are pairwise non-quasi-isometric.

This continuum can be viewed as a Cantor subset X in the space of marked groups with a natural continuous map $T : X \to X$ which preserves many group properties. In fact the dynamical system (X, T) is topologically conjugate to the one sided shift. I will explain why, for any reasonable T-invariant probability measure μ on X, a typical (i.e. μ-almost) property of a group from the family of groups X is to have growth bounded from above by a function of the type $e(n(\alpha))$, where α is a constant smaller that 1.

At the same time a co-meager subset $Y \subset X$ of groups has a different type of behavior at infinity, namely it has the so-called oscillating growth which I will define during the talk. At the beginning of the talk we will also discuss a general approach to randomness in group theory based on the use of the space of marked groups.

Graphical C(6) and C(7) small cancellation groups

Dominik Gruber (Universität Vienna)

Abstract: Small cancellation theory has been a rich source of examples of infinite groups with exotic properties, so-called monsters. Examples include Tarksi monsters, which are non-cyclic groups where every nontrivial proper subgroup is cyclic of order a fixed prime number p, and Gromov's monsters, which are groups that coarsely contain expander graphs in their Cayley graphs.

In my talk, I will explain basic concepts of classical $C(6)$ and $C(7)$ small cancellation theory, which uses the combinatorics of planar 2-complexes to prove facts about groups whose presentations satisfy certain combinatorial conditions.

I will then go into the more recent graphical small cancellation theory introduced by Gromov. Graphical small cancellation theory is a generalization of classical small cancellation theory that allows constructions of groups containing certain prescribed subgraphs in their Cayley graphs.

I will present an elementary combinatorial approach to the theory focusing on the generalizations of the classical $C(6)$ and $C(7)$ small cancellation conditions. I will explain how fundamental results of classical small cancellation theory can be extended to graphical small cancellation presentations.

Moreover, I will discuss new results on infinitely presented graphical $C(7)$ groups, and I will show that this combinatorial approach can be used to construct groups that coarsely contain prescribed infinite sequences of finite graphs. (In this context, "coarsely" is a weakening of "quasi-isometrically".)

Sharply 2-ransitive linear groups

Dennis Gulko (Ben-Gurion University of the Negev)

Abstract: This is a joint work with Yair Glasner. A group G is sharply 2-transitive if it admits a faithful permutation representation that is transitive and free on pairs of distinct points. Conjecturally, for all such groups there exists a near-field N (i.e. a skew field that is distributive only from the left) such that G is isomorphic to the semidirect product of the multiplicative and additive groups of N. This is well known in the finite case. We prove this conjecture when $G < \mathrm{GL}(n, F)$ is a linear group, excluding characteristic 2 in two separate ways.

Stabilizer rigidity in irreducible group actions

Yair Hartman (Weizmann Institute)

Abstract: The celebrated Stuck–Zimmer theorem classifies the action stabilizers (IRSs) of semisimple high rank Lie groups. Bader–Shalom proved the same result for a product of locally compact property (T) groups.

We prove a generalization of these two theorems, where we remove the property (T) requirement. For example, our results apply to $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. The proof uses Poisson boundary theory to establish co-amenability of invariant random subgroups when an intermediate factor theorem applies.

This is joint work with Omer Tamuz.

Extensions of amenable groups by recurrent groupoids

Kate Juschenko (Northwestern University)

Abstract: I will discuss a theorem on amenability which unifies many know technical proofs of amenability to the one common proof as well as produces examples of groups for which amenability was an open problem. This is joint with V. Nekrashevych and M. de la Salle.

Acute triangulations of the sphere

Sang-hyun Kim (KAIST)

Abstract: We prove that a simplicial complex L homeomorphic to a sphere can be realized as an acute geodesic triangulation of a sphere if and only if the right-angled Coxeter group defined by L is CAT(-1). We also study generalizations of this result to other angle bounds, other planar surfaces and other dimensions.

Commensurating endomorphisms of acylindrically hyperbolic groups and applications

Ashot Minasyan (University of Southampton)

Abstract: Two elements x, y of a group G are said to be commensurable if some non-zero power of x is conjugate to some non-zero power of y. An endomorphism f of G is called commensurating if $f(x)$ is commensurable to x for all $x \in G$. In the talk I will discuss the structure of such endomorphisms when G is acylindrically hyperbolic (the extensive class of acylindrically hyperbolic groups has been recently defined by Denis Osin, and includes all non-elementary relatively hyperbolic groups, most mapping class groups of surfaces, outer automorphism groups of free groups, etc.) The main result is that a commensurating endomorphism of an acylindrically hyperbolic group G is necessarily an inner automorphism provided G has no non-trivial finite normal subgroups. This has applications towards the study of the outer automorphism group of G. For instance we show that $\mathrm{Out}(G)$ is residually finite if G is virtually compact special (in the sense of Haglund–Wise). We also prove a similar result when G is the fundamental group of any compact 3-manifold. The talk will be based on a joint work with Yago Antolin and Alessandro Sisto.

On Nielsen equivalence in finitely generated groups

Aglaia Myropolska (Université de Genève)

Abstract: Various problems from group theory, geometry and combinatorics motivate the study of the natural action of the group $\mathrm{Aut}(F_n)$ on the set $\mathrm{Epi}(F_n, G)$ of generating n-tuples in a group G generated by at least n elements. This action was mainly studied in the case when G is finite. We shall consider the situation when G is infinite and after an introduction into the subject we will discuss some new results on transitivity and non-amenability of this action.

Shadowing for group actions

Alexey Osipov (Centro de Giorgi, Pisa)

Abstract: Theory of dynamical systems studies trajectories of group actions on metric spaces. Classical theory of dynamical systems deals with the case of actions of $\mathbb{Z}$ and $\mathbb{R}$. In the first part of the talk we will study mostly the classical case. A dynamical system has a shadowing property if any sufficiently precise approximate trajectory is close to some exact trajectory. We will consider various examples of dynamical systems with and without of the shadowing property. One of the key results of theory of shadowing is a shadowing lemma proved by Anosov and Bowen in 1970s, which claims that a so-called hyperbolic dynamical system has shadowing. We will discuss generalizations of the notion of shadowing and variations of the shadowing lemma for actions of various finitely generated groups. It turns out that they depend on the structure of the group.

Usually an action of a finitely generated group is called hyperbolic if the action of some element is a hyperbolic diffeomorphism. Shadowing lemma for abelian groups was proved by Pilyugin and Tikhomirov in 2003. We will generalize shadowing lemma to actions of nilpotent groups (i.e. prove that any hyperbolic action of a nilpotent group has shadowing). However it turnes out that a large class of “reasonable” actions of large groups (free groups, groups with infinitely many ends) does not have shadowing. So for them the shadowing lemma cannot be formulated in any reasonable way. Finally we will consider particular actions of a solvable Baumslag–Solitar group. It turns out that for them having shadowing dependes on some quantitative characteristics of hyperbolicity of the action. The talk leads us to an interesting open question: for which groups does the shadowing lemma hold?

The talk is based on a joint work with Sergey Tikhomirov.

Graphs and spectral approximation

Felix Pogorzelski (Universität Jena)

Abstract: Our aim is to approximate spectral quantities of operators on discrete structures via finite dimensional analogues. We first introduce Benjamini–Schramm convergent graph sequences which can be seen as a generalization of sofic approximations of finitely generated, sofic groups. Using a construction of Elek, we show how we can assign a graphing to each such sequence which is basically a probability distribution of graphs endowed with a measure preserving group action. To illustrate this concept, some examples will be given. We demonstrate the idea of spectral approximation by considering the spectral distribution functions (integrated density of states) of a random family of bounded, self-adjoint operators defined on the graphing. It is a well known fact that the corresponding eigenvalue counting functions of the graphs in the sequence converge weakly to the integrated density of states of the graphing. A more delicate question is the so-called Lück conjecture which is directly related to the uniform spectral convergence. We explain geometric models for which the uniform convergence can be established and conclude with some open questions on the matter.

PROVING VANISHING OF ℓ^2-COHOMOLOGY WITHOUT ℓ^2

Roman Sauer (Karlsruhe Institute of Technology)

Abstract: We start by reviewing ℓ^2-cohomology and ℓ^2-Betti numbers. After that we explain how a few basic facts about ℓ^2-cohomology combined with homological techniques (as can be found in Ken Brown's book on group cohomology) lead to strong vanishing results, e.g. for universal lattices, Thompson's group V and self-similar groups. The methods are very algebraic in spite of the analytic definition of ℓ^2-Betti numbers. Based on joint work with Uri Bader, Alex Furman, and Werner Thumann.

COCOMPACT LATTICES IN LOCALLY COMPACT KAC–MOODY GROUPS

Anne Thomas (University of Glasgow)

Abstract: A locally compact group G is a group with compatible algebraic, topological and measure-theoretic structures. A connected example is $G = \mathrm{SL}(n, \mathbb{R})$, and a totally disconnected example is $G = \mathrm{SL}(n, K)$, where K is the field of Laurent polynomials over the finite field F_q. A cocompact lattice in a locally compact group G is a discrete subgroup Γ so that the coset space $G \backslash \Gamma$ is compact. We construct the first examples of cocompact lattices in many locally compact Kac-Moody groups G. These groups G are totally disconnected and in general non-linear, but they act "nicely" on an associated simplicial complex called a building. We use this action to construct our examples, which include lattices with surface subgroups and lattices which are free groups. This is joint work with Inna Capdeboscq.