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## *Research Description*

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I am a first-year (and actually even first-month) PhD student at ETH Zurich, under the supervision of Prof. Marc Burger. At this moment, I am interested in studying the problem of the *flat closing conjecture*:

**Conjecture.** *Let  $X$  be a proper CAT(0) space and  $\Gamma$  a discrete group acting geometrically on  $X$ . If  $X$  contains a  $d$ -dimensional flat, then  $\Gamma$  contains a subgroup isomorphic to  $\mathbb{Z}^d$ .*

In my Master thesis, I analysed the case of geodesically complete proper CAT(0) spaces, as treated by P.E. Caprace and G. Zadnik in [1]. As a starting point for my own research in this direction, I have decided to focus on CAT(0) cube complexes, mainly because they are relatively simple and there are many examples that can be placed into this class of spaces. Currently, I am trying to enrich my own collection of examples, and I am reading the papers [2] and [5] to learn what is known about this conjecture in the universe of CAT(0) cube complexes.

At some point of my PhD, I would also like to work on some open problems related to the growth of groups, for example, the gap conjecture on group growth, as presented in [3] and treated in [4] by R. Grigorchuk.

## References

- [1] Caprace, P.-E.; Zadnik, G. Regular elements in CAT(0) groups. (2011) arXiv:1112.4637.
- [2] Caprace, P.-E.; Sageev, M. Rank rigidity for CAT(0) cube complexes. *Geom. Funct. Anal.* 21 (2011), no. 4, 851–891.
- [3] Grigorchuk, R. I. On growth in group theory. *Proceedings of the International Congress of Mathematicians, Vol. I, II* (Kyoto, 1990), 325–338, Math. Soc. Japan, Tokyo, 1991.
- [4] Grigorchuk, R. I. On the gap conjecture concerning group growth. (2012) arXiv:1202.6044v2.
- [5] Sageev, M.; Wise, D.T. Periodic flats in CAT(0) cube complexes. *Algebr. Geom. Topol.* 11 (2011), no. 3, 1793–1820.