

ASYMPTOTIC PROPERTIES OF \mathbb{Z}^n -ACTIONS

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We consider the stable norm associated to a discrete, torsionless abelian group of isometries $\Gamma = \mathbb{Z}^n$ of a geodesic space (X, d) . We show that the difference between the stable norm $\|\cdot\|_{\text{st}}$ and the distance d is bounded by a constant only depending on the rank n and on upper bounds for the diameter of $Y = \Gamma \backslash X$ and the asymptotic volume $\omega(\Gamma, d)$. We also prove that the upper bound on the asymptotic volume is equivalent to a lower bound for the stable systole of the action of Γ on (X, d) ; for this, we establish a Lemma à la Margulis for \mathbb{Z}^n -actions, which gives optimal estimates of the asymptotic volume $\omega(\Gamma, d)$ in terms of $\text{stsys}(\Gamma, d)$, and vice versa, and characterize the cases of equality. As an application in Riemannian geometry, we prove that the number of connected components of any optimal, integral 1-cycle in a closed Riemannian manifold Y either is bounded by an explicit function of the first Betti number, $\text{diam}(Y)$ and $\omega(H_1(Y, \mathbb{Z}))$, or is a sublinear function of the mass.