BETTI NUMBERS OF TOWERS OF COVERS OF REFLECTION GROUPS

Michael Davis

Fifteen years ago Boris Okun and I had an idea about how to use Lück's Approximation Theorem to prove the vanishing of L^2-Betti numbers of certain right-angled Coxeter groups W. We had in mind a specific tower of covers by manifolds $\to M_i \to M_{i-1} \to$ as well as a formula for the ordinary Betti numbers $b_k(M_i)$. However, as $i \to \infty$ the ratios $b_k(M_i)/[W : \pi_1(M_i)]$ usually did not limit to 0 even in cases where the L^2-Betti numbers of W were known to vanish. Why didn't this contradict Lück's Theorem? Eventually we realized that the reason was that the covers were not regular, i.e., the subgroups $\pi_1(M_i)$ were not normal.