

Expanders, fixed point properties on Banach spaces and random graphs

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Finite quotients of groups with Kazhdan's property (T) are among the first known examples of expanders. Property (T) is satisfied for instance by lattices in simple groups of rank at least two, and in some simple groups of rank one, and by random groups.

Several strengthened versions of property (T) in the setting of Banach spaces have been formulated in recent years, each of them interesting for various reasons: relevance for the conjectures of Baum-Connes and Novikov, separation between rank one and higher rank, examples of expanders without coarse embedding into any uniformly convex Banach space, presumed connection to the conformal dimension of the boundary, stronger rigidity results (e.g. requiring weaker conditions of smoothness) etc.

In this talk I shall explain a unified approach to property (T) in the setting of Banach spaces, using random walks. An application of this approach in the context of chains of finite index subgroups yields a Banach version of property (tau), consistent with a Banach version of expanders. I shall then explain how, surprisingly, random graphs have expansion properties stronger than the classical ones, and therefore random groups have stronger versions of property (T). This is based on joint work with J. Mackay and P. Nowak.