COVERINGS AND EXPANDERS

Emmanuel Kowalski

Expander graphs have been used in connection with the study of geometric properties of towers of coverings of Riemannian manifolds for a long time, in particular by Buser, Burger and Brooks.

In recent years, new applications of expanding properties of families of Cayley and Schreier graphs related to such towers have appeared, which go beyond the previously known relations between combinatorial and Riemannian Laplace eigenvalues:

(1) Lackenby has related the Heegard genus of 3-manifolds to isoperimetric constants (or Laplace eigenvalues), which shows for instance that the Heegard genus of arithmetic 3-manifolds grows with their volume;

(2) Gromov and Guth have shown, among other things, that the knots that arise as the ramification locus of ramified coverings of towers of 3-manifolds have large "distorsion", as defined by Gromov: whenever they are embedded in space, there are points "far away on the knot" which are "close" in space; this provides a large family of examples of knots with this property (the existence of which was first proved by Pardon);

(3) Ellenberg, Hall and Kowalski showed very strong relations between the finiteness of algebraic points of bounded degree on towers of algebraic curves and the expanding properties of the associated graphs, with many diophantine consequences.

These new developements are sometimes very surprising. They are, at least in part (and very strikingly for the number-theoretic applications), linked to the considerable progress of our understanding of which families of Cayley graphs of Zariski-dense subgroups of arithmetic groups form expanders, especially influenced by the work of Helfgott and Bourgain–Gamburd.

The course will review the definitions and basic properties of expander graphs. It will then present some of these recent geometric and arithmetic applications, focusing on some analogies that appear in the flow of the arguments, despite their apparent diversity. The ideas of the proof of expansion for Cayley graphs will also be discussed.