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HIGHER TEICHÜLLER THEORY

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The Teichmüller space of a surface S is an important object in mathematics and physics. It has many facets, and carries many interesting structures itself. In particular, it can be seen as a connected component of (conjugacy classes of) representations from the fundamental group of S into $\mathrm{PSL}(2,\mathbf{R})$, consisting entirely of discrete and faithful representations. Generalizing this point of view, Higher Teichmüller Theory studies connected components of (conjugacy classes of) representations from the fundamental group of S into more general semisimple Lie groups which consist entirely of discrete and faithful representations. Important families of examples have been discovered, such as the Hitchin component of representations into $\mathrm{PSL}(n,\mathbf{R})$ (or more generally into a split real simple Lie group), or components of maximal representations into $\mathrm{Sp}(2n,\mathbf{R})$ (or more generally into a simple Lie group of Hermitian type). Such examples have been shown to share striking features with classical Teichmüller space. For instance, in the same way that classical Teichmüller space parametrizes all possible finite-volume hyperbolic structures on S, the Hitchin component of representations into $\mathrm{PSL}(3,\mathbf{R})$ parametrizes all possible convex real projective structures on S. One of the goals of Higher Teichmüller Theory is precisely to find interesting geometric structures parametrized by connected components of representations in a very general setting.

We will give a survey of some aspects of Higher Teichmüller Theory, which is a very active and quickly expanding area of research. In particular, we will provide a number of concrete examples. We will consider discrete and faithful representations, not only of surface groups, but more generally of Gromov hyperbolic groups, and will make links with the recent powerful notion of Anosov representation.