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MODULI SPACES
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MODULI SPACES OF GRAPHS

Karen Vogtmann
University of Warwick

Finite metric graphs parametrize many phenomena in mathematics and science, and the goal of this minicourse is to investigate the topology and geometry of the parameter spaces, i.e. of moduli spaces of graphs. Geometric group theory provides a way to investigate such spaces by viewing them as quotients of contractible spaces with proper actions by appropriate groups. There are natural ways to modify these spaces to obtain cocompact proper actions as well. Thus one can transfer information between algebraic properties of the groups and geometric and topological properties of the spaces. (If the graphs have no leaves and fixed fundamental group \mathbb{F}_n , the relevant group is the group $\text{Out}(\mathbb{F}_n)$ of outer automorphisms of a finitely-generated free group, and the relevant moduli space is known as Outer space.)

We will investigate the topology of moduli spaces of graphs with or without leaves, using both geometric and algebraic methods. In particular we will explain the connection, discovered by Kontsevich, with the symplectic derivation algebra of a free Lie algebra, show how to assemble cycles in spaces of graphs with leaves to obtain cycles for graphs of higher rank, explain how modular forms enter the picture, and show how to perform Euler characteristic calculations which show that there is a vast amount of as yet unexplained homology lurking in these spaces.