High-dimensional cohomology of $\mathbf{Sp}_{\boldsymbol{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

High-dimensional cohomology of $\mathrm{Sp}_{2n}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Facts:

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Facts:

1. $\mathrm{Sp}_{2n}(\mathbb{Z})$ is a *virtual duality group*, i.e. there are isomorphisms

$$H^{n^2-i}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong H_i(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathrm{St}_n^{\omega} \otimes \mathbb{Q})$$

   where St_n^{ω} is the *dualising module* (Steinberg module)

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Facts:

1. $\mathrm{Sp}_{2n}(\mathbb{Z})$ is a *virtual duality group*, i.e. there are isomorphisms

$$H^{n^2-i}(\mathrm{Sp}_{2n}(\mathbb{Z}),\mathbb{Q}) \cong H_i(\mathrm{Sp}_{2n}(\mathbb{Z}),\mathrm{St}_n^{\omega}\otimes\mathbb{Q})$$

   where St_n^{ω} is the *dualising module* (Steinberg module)

2. $\mathrm{St}_n^{\omega} \cong H_{n-1}(\Delta_n)$, the top-dimensional homology the Tits building Δ_n associated to $\mathrm{Sp}_{2n}(\mathbb{Z})$.

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Facts:

1. $\mathrm{Sp}_{2n}(\mathbb{Z})$ is a *virtual duality group*, i.e. there are isomorphisms

$$H^{n^2-i}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong H_i(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathrm{St}_n^{\omega} \otimes \mathbb{Q})$$

   where St_n^{ω} is the *dualising module* (Steinberg module)

2. $\mathrm{St}_n^{\omega} \cong H_{n-1}(\Delta_n)$, the top-dimensional homology the Tits building Δ_n associated to $\mathrm{Sp}_{2n}(\mathbb{Z})$.

→ Understanding $\mathrm{St}_n^{\omega} \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ - module allows to compute high-dimensional rational cohomology of $\mathrm{Sp}_{2n}(\mathbb{Z})$.

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Fact: $H^{n^2-i}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong H_i(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathrm{St}_n^{\omega} \otimes \mathbb{Q})$

Aim:

Obtain a presentation of $\mathrm{St}_n^{\omega} \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module.

Hope:

This presentation can be used to show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Aim: Obtain a presentation of $\mathrm{St}_n^\omega \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module;

show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Aim: Obtain a presentation of $\mathrm{St}_n^\omega \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module;

show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

Why and how should this work?

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Aim: Obtain a presentation of $\mathrm{St}_n^\omega \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module; show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

Why and how should this work?

Gunnells:
St_n^ω is generated by „integral apartment classes“. This implies

$$H^{n^2}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$$

High-dimensional cohomology of $\mathbf{Sp}_{2n}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Aim: Obtain a presentation of $\mathrm{St}_n^\omega \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module;

show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

Why and how should this work?

Gunnells:
St_n^ω is generated by „integral apartment classes“. This implies

$$H^{n^2}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$$

Church-Farb-Putman:
Analogous work in the case of $\mathrm{SL}_n(\mathbb{Z})$.

High-dimensional cohomology of $\mathbf{Sp_{2n}}(\mathbb{Z})$

Benjamin Brück, work in progress w. *Peter Patzt & Robin Sroka*

Aim: Obtain a presentation of $\mathrm{St}_n^\omega \cong H_{n-1}(\Delta_n)$ as an $\mathrm{Sp}_{2n}(\mathbb{Z})$ -module; show that $H^{n^2-1}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$

Why and how should this work?

Gunnells:
St_n^ω is generated by „integral apartment classes". This implies

$$H^{n^2}(\mathrm{Sp}_{2n}(\mathbb{Z}), \mathbb{Q}) \cong 0$$

Church-Farb-Putman:
Analogous work in the case of $\mathrm{SL}_n(\mathbb{Z})$.

Geometric argument.