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ON COUNTING, WITH NUMBER THEORETIC FLAVORS

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Counting and distribution questions pervade every field of mathematics. These very different motivations lead to very different tools. Hopefully, in settings sharing sufficiently many common features, these different ap- proaches can complement, and even inform, one another. This is spectacularly true of problems involving a (nice) underlying group action; through it, we are granted access to automorphic forms, ergodic theory, hyperbolic geometry, representation theory, Teichmüller or homogeneous dynamics...

In this mini-course, we will illustrate some recurring themes and techniques at this rich intersection of topics, with the accent on specific contributions of, and respectively to, number theory. We will introduce key concepts and illustrate their workings at the hand of the following examples.

(1) How a certain rate of equidistribution for periodic orbits of the horocyclic flow is equivalent to the Riemann Hypothesis (Zagier, Sarnak 1981),
(2) How counting closed geodesics on a closed hyperbolic surface, ordered by length, mimics the large scale distribution of prime numbers (Delsarte, Huber, Selberg, 1950s),
(3) How Fourier coefficients of cups forms for a lattice Γ are multiplicative if and only if Γ is arithmetic (Venkatesh, 2000s).