A growth characterization of Property (T)

based on a joint work with R. Dougall, B. Schapira and S. Tapie

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Property (T) vs. Random walks

- $Q = F(S)/N$ a finitely generated group
- μ a symmetric probability measure supported by $S \cup S^{-1}$.
- $\pi\colon Q \to \mathcal{U}(\mathcal{H})$ a unitary representation.

Markov operator:

$$M_\pi = \sum_{g \in G} \mu(g)\pi(g), \qquad M_\pi \in \mathcal{B}(\mathcal{H}), \qquad \|M_\pi\| \leqslant 1$$

Theorem

Q has property (T) if and only if there exists $\varepsilon > 0$ such that for every unitary representation $\pi\colon Q \to \mathcal{U}(\mathcal{H})$ without non-zero invariant vectors,

$$\rho(M_\pi) \leqslant 1 - \varepsilon$$

From random walk to geodesic flow

$p\colon G \twoheadrightarrow Q$ a projection where G is a hyperbolic group

Exponential growth rate

h_G: critical exponent of the *Poincaré series* $\mathcal{P}(s) = \sum_{g\in G} e^{-s|g|}$

$\pi\colon Q \to \mathcal{U}(\mathcal{H})$ *positive* unitary representation

Twisted Poincaré series $A(s) \in \mathcal{B}(\mathcal{H})$

$$A(s) = \sum_{g\in G} e^{-s|g|} \pi \circ p(g)$$

h_π its critical exponent.

Philosophy

$$h_\pi - h_G \leftrightsquigarrow \log \rho(M_\pi)$$

Main result

$p\colon G \twoheadrightarrow Q$ a projection where G is a hyperbolic group

Theorem

Q has property (T) if and only if there exists $\varepsilon > 0$ such that for every unitary representation $\pi\colon Q \to \mathcal{U}(\mathcal{H})$ without non-trivial finite dimensional subrepresentation,

$$h_{\pi\otimes\bar{\pi}} \leqslant h_G - \varepsilon$$