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ERGODIC THEOREMS, MULTIPLE MIXING, AND ARITHMETIC COUNTING PROBLEMS

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A natural approach for studying asymptotic properties of group actions is to analyze the behavior of the corresponding averaging operators along orbits. This leads to a number of interesting analytic and representation-theoretic questions. Although such operators behave erratically on finite scales, they have a limit as the scale tends to infinity leading to the ergodic theorems. In the first part of the mini-course, we develop this theory, and then explain how it could be used to address the lattice counting problems. In particular, we explore the questions of counting the number integral points lying on group varieties and contained in a large ball and the questions of counting the number of rational points satisfying Diophantine inequalities. It turns out that the error terms in these counting problems are intimately related to the spectral-gap bounds for the corresponding automorphic representations. We discuss the property tau of the automorphic representations and indicate how to establish the counting estimates uniformly over congruence conditions, which is important for number-theoretic applications. In the second part of the mini-course, we will discuss estimates on correlations for homogeneous dynamical systems that provide a natural way to quantify independence of observables generated by group actions. We explain how these estimates can be used to study statistical properties of counting functions. Surprisingly, we will see that some deterministic counting functions behave similarly to sums of independent random variables, for instance, we prove that they satisfy the central limit theorem and other probabilistic limit laws.