Uniform Exponential Growth in Negative curvature

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1. w/ Kropholler, Lyman
2. w/ Abbott, Gupta, Petyt, Spriano

Counting Problems
Ventotene 2019

GROWTH — COUNTING SIZE OF BALLS

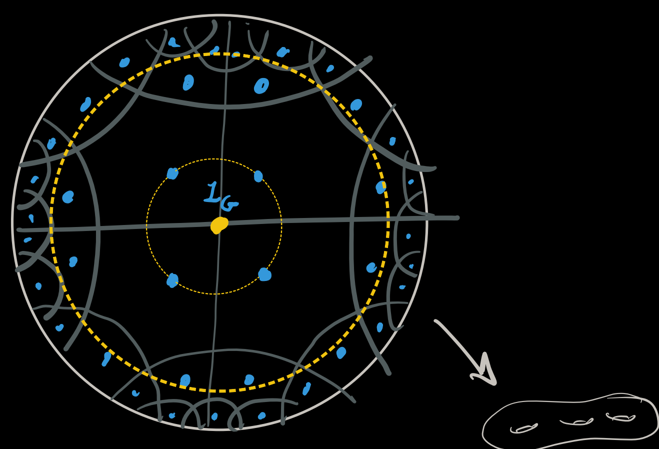
G: (infinite) group,

A: arbitrary FINITE generating set.

Count elements w/ $\|g\|_A \leq N$. $\boxed{\beta_{G,A}(N)}$

GROWTH of G

$G = \pi_1(\Sigma) \circlearrowright$



X

$\Sigma = X/G$

Theme: Geometry of X influences algebra of G

Computing $\beta_{G,A}(N)$ is CHALLENGING!

Exponential Growth

Def G has <u>exponential growth</u> when

$$w(G,A) := \lim_{n\to\infty} \frac{\ln(\beta_{G,A}(n))}{n} > 0$$

Q (Gromov): When G has exponential growth,

$$\underbrace{w(G,A) > c > 0}_{\text{uniform exponential growth}}$$

independent of generating set

A (Wilson): <u>No</u>

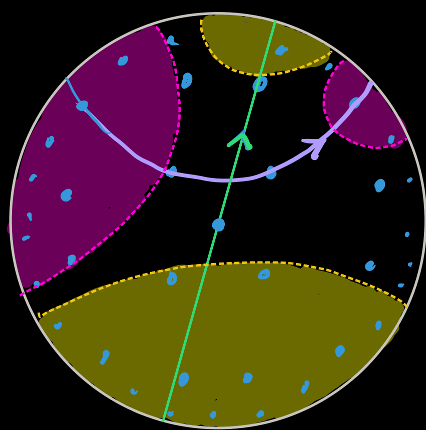
There is a group with exponential growth and not uniform exponential growth

— example is <u>not</u> finitely presented —

BOUNDING SIZE OF BALLS

$G^{f.p.} < Aut(X)$ often has exponential growth.

"negatively curved"



CRITERIA:

Find $a, b \in G$ with uniformly short word length such that $\langle a, b \rangle = F_2$

$\Longrightarrow$ G has uniform exponential growth.

RESULTS

Thm 1 (w/ Kropholler, Lyman): Let G be one-ended, δ-hyperbolic. then $\mathrm{Aut}(G)$ has locally uniform exponential growth.

Thm 2 (w/ Abbott, Gupta, Petyt, Spriano): Let X be a CAT(0) cube complex with a factor system. $G \curvearrowright X$ proper + cocompact then G has uniform exponential growth.

Thank You!