Counting subgroups points

Joint with M. Cordes, J. Russell and A. Zalloum

Davide Spriano

University of Oxford

Setting:

Fix ambient group G, Cayley graph Γ, and a subgroup $H < G$.

Setting:

Fix ambient group G, Cayley graph Γ, and a subgroup $H < G$.

Goal:

Count how many elements of H are in a ball of radius n.

$$\#\{B_n(1) \cap H\}$$

Goal:

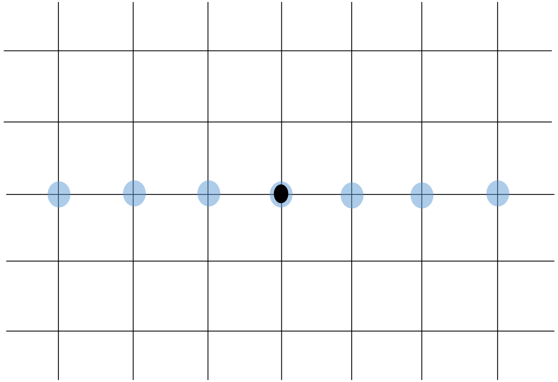
Count how many elements of H are in a ball of radius n.

$$Gr(n):\ \ n \to \#\{B_n(1) \cap H\}$$

Goal:

Count how many elements of H are in a ball of radius n.

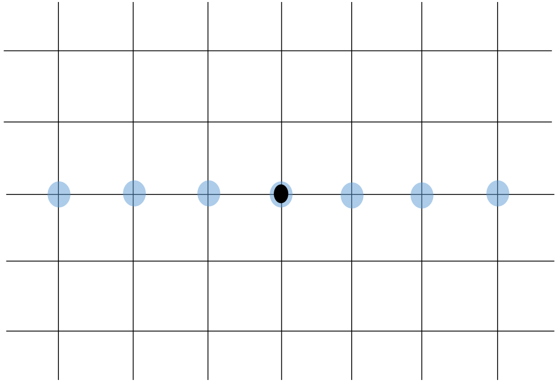
$$Gr(n):\ \ n \to \#\{B_n(1) \cap H\}$$



Goal:

Count how many elements of H are in a ball of radius n.

$$Gr(n):\ \ n \to \#\{B_n(1) \cap H\}$$



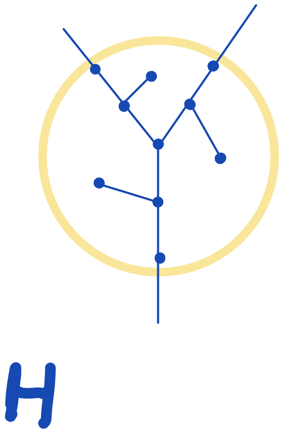
$$Gr_{\mathbb{Z}^2,\mathbb{Z}}(n) = 2n - 1$$

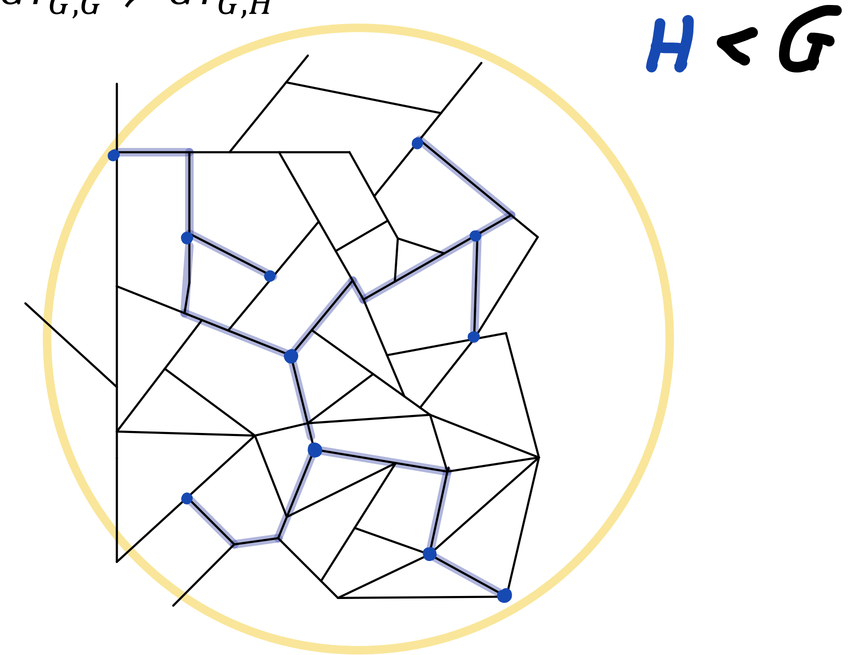
(Asymptotic) comparisons:

- $Gr_{G,G}(n) \geq Gr_{G,H}(n) \Rightarrow Gr_{G,G} \succcurlyeq Gr_{G,H}$
- $Gr_{G,H} \succcurlyeq Gr_{H,H}$

(Asymptotic) comparisons:

- $Gr_{G,G}(n) \geq Gr_{G,H}(n) \Rightarrow Gr_{G,G} \succcurlyeq Gr_{G,H}$
- $Gr_{G,H} \succcurlyeq Gr_{H,H}$

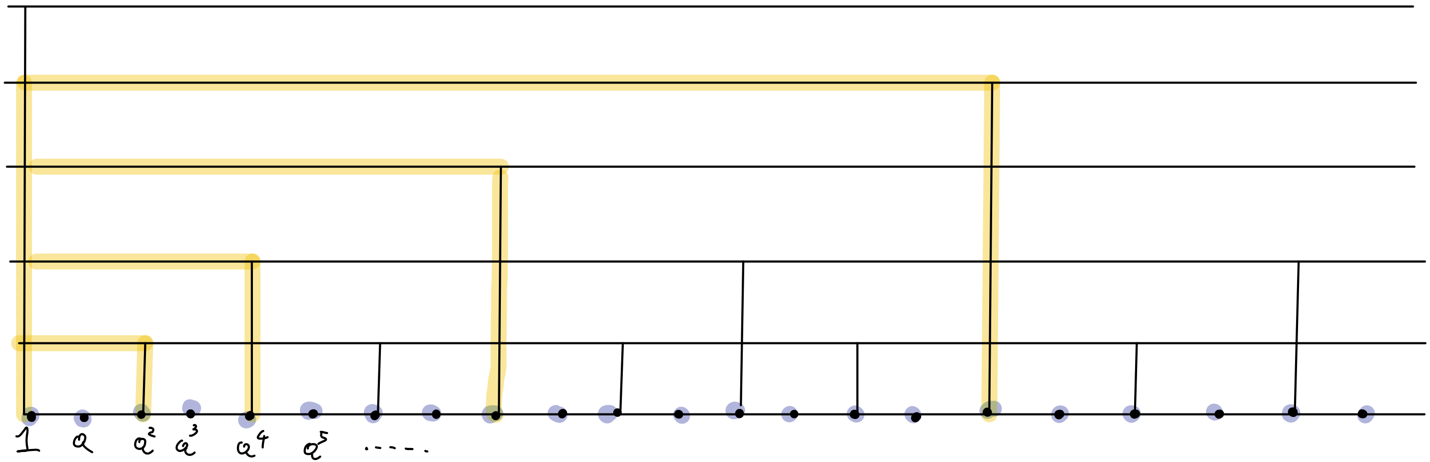




(Asymptotic) comparisons:

- $Gr_{G,G}(n) \geq Gr_{G,H}(n) \Rightarrow Gr_{G,G} \succcurlyeq Gr_{G,H}$
- $Gr_{G,H} \succcurlyeq Gr_{H,H}$

$d(1, a^n) \approx \log(n)$



(Asymptotic) comparisons:

- $Gr_{G,G} \succcurlyeq Gr_{G,H} \succcurlyeq Gr_{H,H}$

(Asymptotic) comparisons:

- $Gr_{G,G} \succcurlyeq Gr_{G,H} \succcurlyeq Gr_{H,H}$

Fact: If H is (non elementary) hyperbolic, $Gr_{H,H}$ is exponential

(Asymptotic) comparisons:

- $Gr_{G,G} \succcurlyeq Gr_{G,H} \succcurlyeq Gr_{H,H}$

Fact: If H is (non elementary) hyperbolic, $Gr_{H,H}$ is exponential

Question: What is the relation between $G_{G,H}$ and $Gr_{G,G}$?

Focus: stable subgroups

Def: H **stable** if 1) hyperbolic and

2) Morse property.

Focus: stable subgroups

Def: H **stable** if 1) hyperbolic and

2) Morse property.

Thm [Dahmani-Futer-Wise]:

H stable, G hyperbolic. Then $Gr_{G,H}$ is exponentially smaller than $Gr_{G,G}$.

Our result

Thm [Cordes-Russell-S-Zalloum]:

H stable, G MLTG. Then $Gr_{G,H}$ is exponentially smaller than $Gr_{G,G}$.

Our result

Thm [Cordes-Russell-S-Zalloum]:

H stable, G MLTG. Then $Gr_{G,H}$ is exponentially smaller than $Gr_{G,G}$.

MLTG:- (Hyperbolic)

- Mapping Class Groups (HHGs)
- CAT(0)
- $\pi_1(M^3)$
- …

Thank you for your attention!