

**VENTOTENE INTERNATIONAL WORKSHOPS V
COUNTING PROBLEMS
VENTOTENE, 6-11 SEPTEMBER 2021**

FLAT AND HYPERBOLIC ENUMERATIVE GEOMETRY

Anton Zorich

Université Pierre et Marie Curie-Université Paris Diderot

This course is devoted to the count of simple closed hyperbolic geodesics (following Maryam Mirzakhani) and to the count of square-tiled surfaces. We shall see that the two counts are, actually, closely related. We shall discuss certain tools from dynamics (like ergodicity of the group action on the space of measured laminations) and certain tools from complex geometry (like intersection theory of the moduli space of curves). Both tools are particularly useful in both counts. We also discuss certain relations and application to combinatorics (like count of integral metric ribbon graphs following Maxim Kontsevich and count of meanders).

Square-tiled surfaces can be interpreted as integer points in the moduli spaces of Abelian and quadratic differentials. This interpretation allows to define the Masur–Veech volume of these moduli spaces in terms of the asymptotic count of square-tiled surfaces. One can also interpret volume contributions of square-tiled surfaces carrying specified combinatorial geometry, as "frequencies" of square-tiled surfaces having given geometry. The resulting frequencies describe statistical geometry of a "random" square-tiled surfaces. These frequencies are closely related to Kontsevich's count of metric ribbon graphs and coincide with Mirzakhani's frequencies of simple closed hyperbolic multicurves.

We shall discuss the recent progress in these problems and finish by a conjectural description of a "random" square-tiled surface and of a "random" simple geodesic multicurve on a surface of large genus.