Λ-trees and the LEAN THEOREM PROVER

Raphael Appenzeller
Ventotene
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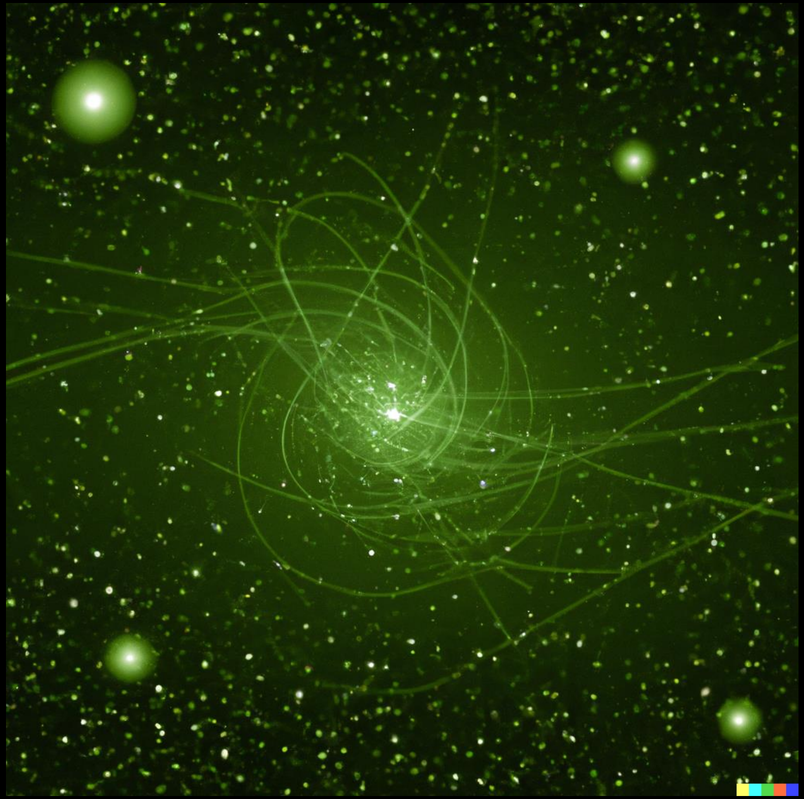

DALL-E AI result for the prompt:
`Λ-metric space`

Λ-trees

Let Λ be an ordered abelian group.

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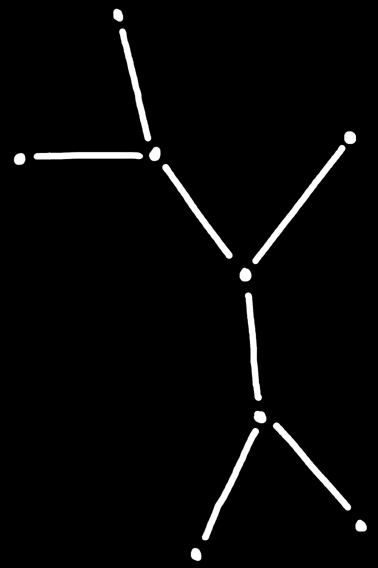
$$\Lambda = \mathbb{Z}$$



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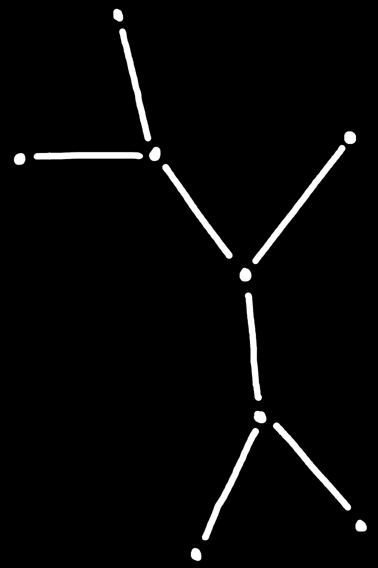
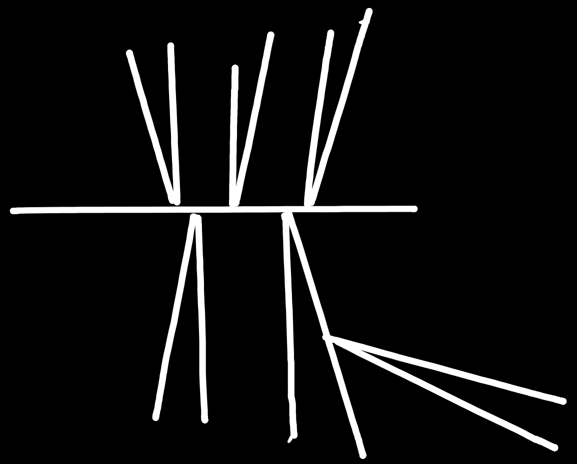


$\Lambda = \mathbb{R}$



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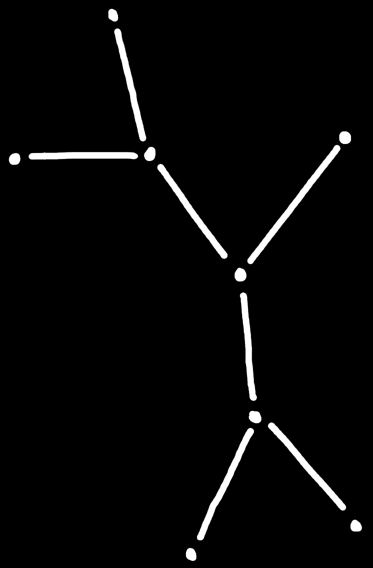

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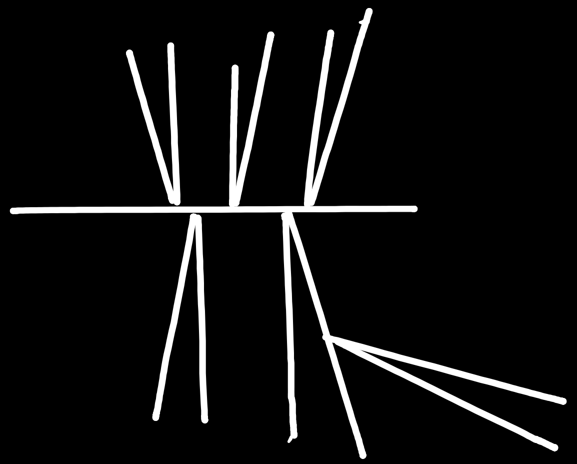
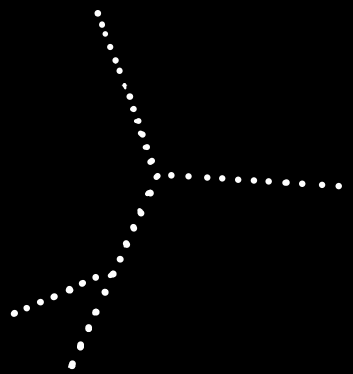


$\Lambda = \mathbb{Q}$



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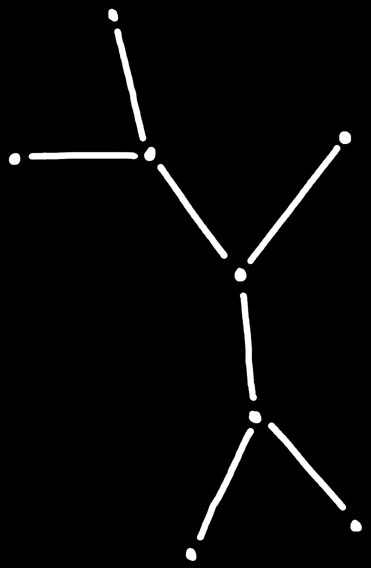

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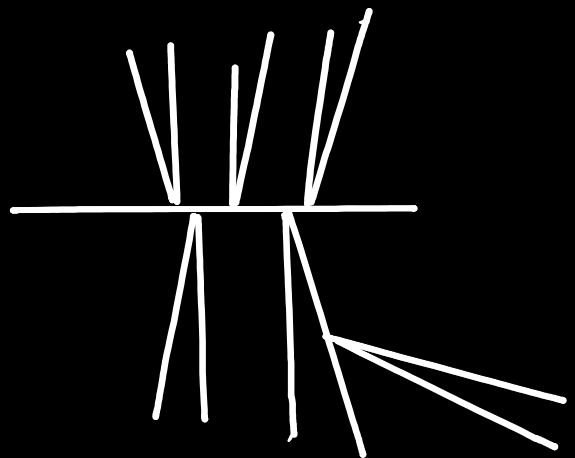

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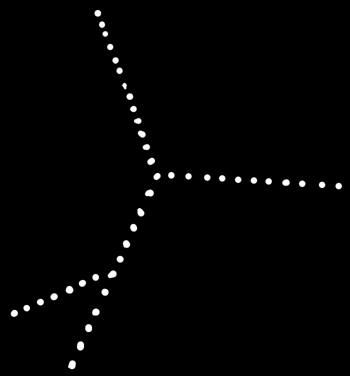
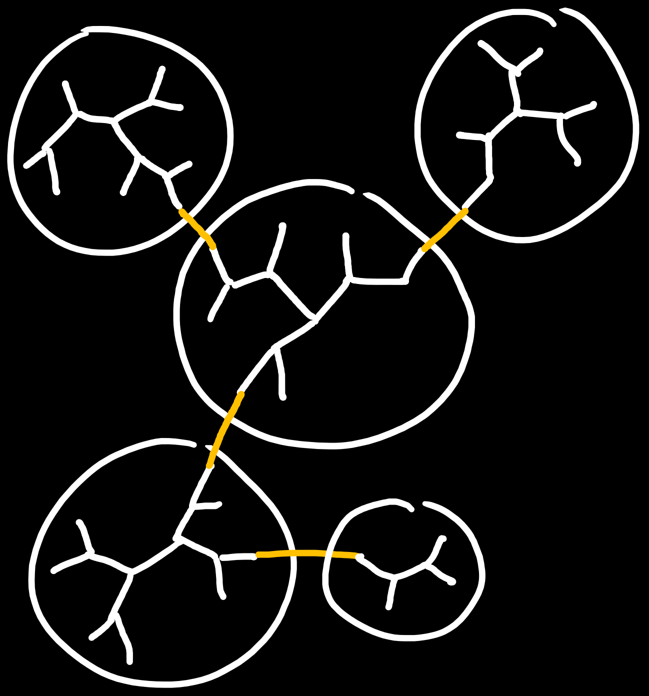


$\Lambda = \mathbb{Z} \times \mathbb{Z}$



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Def: A Λ-metric space is a Λ-*tree* if

(1) connected

(2) no cycles

(3) no holes

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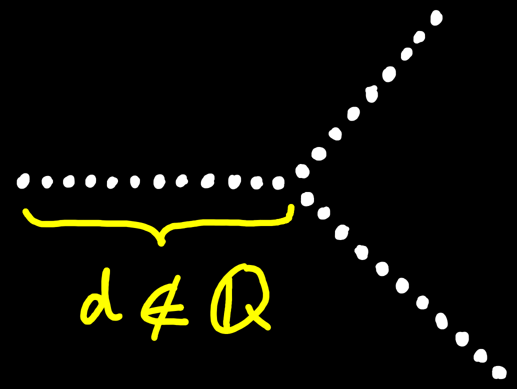
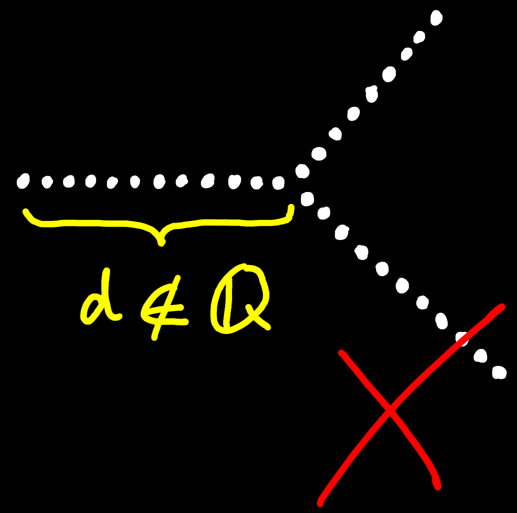
Example: $\Lambda = \mathbb{Q}$



Λ-trees

Example: $\Lambda = \mathbb{Q}$



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Theorem: If $\Lambda = 2\Lambda$, then $(1) \wedge (2) \implies (3)$.

LEAN THEOREM PROVER

LEAN is a formal proof verifier

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```
class lambda_tree (Λ : Type*)[linear_ordered_add_comm_group Λ]
                  (X : Type*) extends lambda_metric_space Λ X :=
(a1 : axiom_1 Λ X)
(a2 : axiom_2 Λ X)
(a3 : axiom_3 Λ X)
```

LEAN

THEOREM PROVER

Theorem: If $\Lambda = 2\Lambda$, then $(1) \wedge (2) \implies (3)$.

```
theorem thm1 (Λ : Type*)[linear_ordered_add_comm_group Λ]
             (X : Type*)[lambda_metric_space Λ X] :
Lambda_is_two_Lambda Λ → axiom_1 Λ X ∧ axiom_2 Λ X → axiom_3 Λ X
```

THEOREM PROVER

Theorem: If $\Lambda = 2\Lambda$, then $(1) \wedge (2) \implies (3)$.

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```

goals accomplished 🎉