Simplicial volume of manifolds with amenable π_1^∞

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Definition (Gromov, 1982)

$$\|M\| := \inf\{\|c = \sum a_i\sigma_i\|_1 |\ [c] = [M]_{\mathbb{R}} \in H_n^{lf}(M,\mathbb{R})\}.$$

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Finiteness criterion (Löh, 2007)

If $M \cong N \setminus \partial N$, and $\pi_1(\partial N)$ is amenable, then $\|M\| < +\infty$.

Fundamental group at infinity

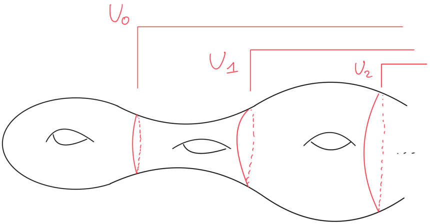

Figure: Nested neighborhoods of infinity.

$$\pi_1^\infty(M) := \varprojlim \pi_1(U_i).$$

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Let M^n be an open n-manifold, with $n \neq 1, 4$.

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Theorem (B., 2022)

Let M^n be an open n-manifold, with $n \neq 1, 4$.

- If M is inward tame and $\pi_1^\infty(M)$ is amenable then $||M|| < +\infty$.
- If M has finitely many ends and is simply connected at infinity then $||M|| < +\infty$.

Thank you for the attention!