Hyperlinearity versus flexible Hilbert Schmidt stability for property (T) groups

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Stability: Rigidity of Approximate Representations

Definition

For a matrix $A \in M_n(\mathbb{C})$, the ***normalized Hilbert Schmidt norm*** *is* $\|A\|_{HS}^2 = \frac{1}{n} Tr(A^*A) = \frac{1}{n}\sum_{i,j} |A_{i,j}|^2$

Definition (Becker, Lubotzky)

A group Γ is ***very flexibly HS-stable*** *if for every given sequence of* ***maps*** $\varphi_n : \Gamma \to U(d_n)$ *such that for all $g, h \in \Gamma$:*

$$\lim_n \|\varphi_n(g)\varphi_n(h) - \varphi_n(gh)\|_{HS} = 0,$$

there exists a sequence of ***true representations*** $\pi_n : \Gamma \to U(D_n)$ *for some $D_n \geq d_n$ such that for all $g \in \Gamma$: $\lim_n \|\varphi_n(g) - P_n\pi_n(g)P_n\|_{HS} = 0$, where $P_n : \mathbb{C}^{D_n} \to \mathbb{C}^{d_n}$ is the projection onto the first d_n-coordinates.*

Stability and Property (T)

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Theorem (D. 2022)

For g ≥ 2, consider the higher rank lattice $\mathrm{Sp}_{2g}(\mathbb{Z})$ and its central extension:

$$1 \longrightarrow \mathbb{Z} \longrightarrow \widetilde{\mathrm{Sp}_{2g}(\mathbb{Z})} \longrightarrow \mathrm{Sp}_{2g}(\mathbb{Z}) \longrightarrow 1,$$

If *$\mathrm{Sp}_{2g}(\mathbb{Z})$ is very flexibly HS-stable, then $\widetilde{\mathrm{Sp}_{2g}(\mathbb{Z})}$ is* ***not*** *hyperlinear (in particular, not sofic).*

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Remark

Same thing happens for many other property (T) groups! (e.g. Gromov random groups). Hilbert-Schmidt Analog of a result of Bowen & Burton.

Thank you for listening!

References:

- Oren Becker & Alex Lubotzky: Group stability and Property (T), Journal of Functional Analysis, 2020.
- Lewis Bowen & Peter Burton: Flexible stability and nonsoficity, Transactions of the American Mathematical Society, 2020.
- Alon Dogon: Flexible Hilbert-Schmidt stability versus hyperlinearity for property (T) groups, arXiv preprint, 2022, avialable at `https://arxiv.org/abs/2211.10492`.