# A model for random (hyp.) 3-manifolds

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# Random manifolds

$$\{ \text{ Set of manifolds } \} + \{ \text{ Probability measure } \} = (\Omega, \mathbb{P})$$

$\Rightarrow$ What is the probability that a random manifold has a certain property?

# Construction models in 3D

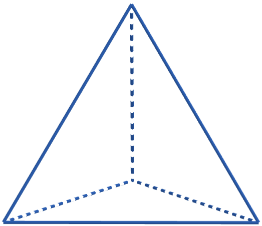
The principal models of construction of random manifolds in 3 dimensions are:

1. Random Heegaard Splittings.
2. Random mapping tori.
3. **Random triangulation**.

# Random triangulation

**General idea**: To construct manifolds by randomly gluing polyhedra together along their faces.

1st attempt:



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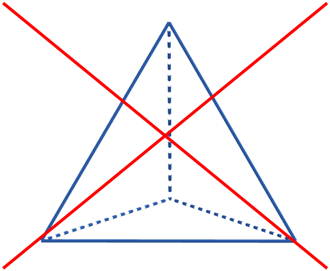
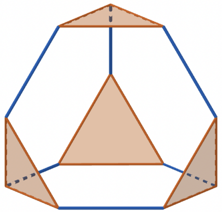


This doesn't work!

The neighbourhoods of the vertices are not typically homeomorphic to $\mathbb{R}^3$.

# Random triangulation

Solution:



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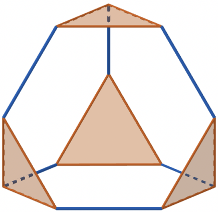


By gluing them along their hexagonal faces **uniformly at random**, we obtain:

$\Rightarrow$ A compact 3-manifold with boundary $\boldsymbol{M_N}$,

where $N$ is the number of tetrahedra.

# Hyperbolic 3-manifolds

**Theorem (Petri-Raimbault, 20)**
$\mathbb{P}[M_N$ *is hyperbolic with totally geodesic boundary*$] \overset{N\to\infty}{\longrightarrow} 1$.

$\Downarrow$ Mostow rigidity

Its geometry can be understood from the combinatorics of the gluing.

# The length spectrum

$$L \longrightarrow C_L(M_N) = \#\{\text{closed geodesics of length} \leq L \text{ on } M_N\},$$

## Theorem (Work in progress)

*As $N \to \infty$, $C_L(M_N)$ converges in distribution to a Poisson random variable $C_L(M)$ with explicit parameter $\lambda(L)$.*

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**THANK YOU!**