

# Actions of higher rank groups on uniformly convex Banach spaces

Tim de Laat

GRAZP: Groups and Rigidity Around the Zimmer Program

September 11, 2023

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Higher rank algebraic/Lie groups and lattices have  $F_{L^p}$  whenever  $1 < p < \infty$ . Examples:  $SL(n, \mathbb{R})$ ,  $SL(n, \mathbb{Q}_p)$ ,  $SL(n, \mathbb{Z})$  for  $n \geq 3$ .

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## Conjecture (Bader–Furman–Gelder–Monod 2005)

Higher rank algebraic/Lie groups and lattices have  $F_E$  for every uniformly convex Banach space  $E$ .

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**The conjecture is true!**