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COCYCLES, BOUNDED COHOMOLOGY AND MILNOR–WOOD INEQUALITIES

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In this talk we will review some classical applications of bounded cohomology to so called Milnor-Wood inequalities, starting with Milnor's celebrated 1958 inequality. A particular emphasis will be given on the usefulness of finding explicit cocycles.

Such an explicit cocycle for the volume of hyperbolic 3-manifolds can be expressed in terms of Bloch–Wigner's dilogarithm. We will see how to exploit cohomological properties to see that this dilogarithm is, up to a constant, the unique measurable function $f : \mathbb{C} \setminus \{0,1\} \to \mathbb{R}$ satisfying the so called 5-terms relation

$$f(x) - f(y) + f\left(\frac{y}{x}\right) - f\left(\frac{1-y}{1-x}\right) + f\left(\frac{x}{y} \cdot \frac{1-y}{1-x}\right) = 0.$$

For smooth functions it is easy to show this with a few derivations.

We will discuss generalizations (joint work with Alessio Savini) of such results and establish new cases of a conjecture on bounded cohomology by Nicolas Monod from 2004.