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ARITHMETICITY, SUPERRIGIDITY, AND TOTALLY GEODESIC SUBMANIFOLDS

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In the 1970's, seminal work of Margulis showed that higher rank lattices have superrigid representations, which in particular implies that all such lattices are arithmetic. Since then, Gromov-Piatetski-Shapiro and Deligne-Mostow have shown that a similar superrigidity theorem cannot hold for all lattices in the isometry group of real or complex hyperbolic space, i.e., in the rank 1 setting. However in work joint with Bader, Fisher, and Stover we show that one can prove certain superrigidity/arithmeticity theorems provided the associated manifold satisfies the geometric condition that it contains infinitely many (maximal) totally geodesic submanifolds. Specifically, we show that this latter criteria forces a hyperbolic manifold to be arithmetic.

The goal of this mini-course will be first to recount the work of Margulis on superrigidity of higher rank lattices and then go on to discuss the proof of the aforementioned theorem. This will include a discussion of the connections between homogeneous dynamics and geodesic submanifolds, their interaction with superrigidity, and an introduction to techniques introduced by Bader and Furman for studying algebraic representations of ergodic actions. Along the way, we will also do our best to discuss remaining open questions in the field.