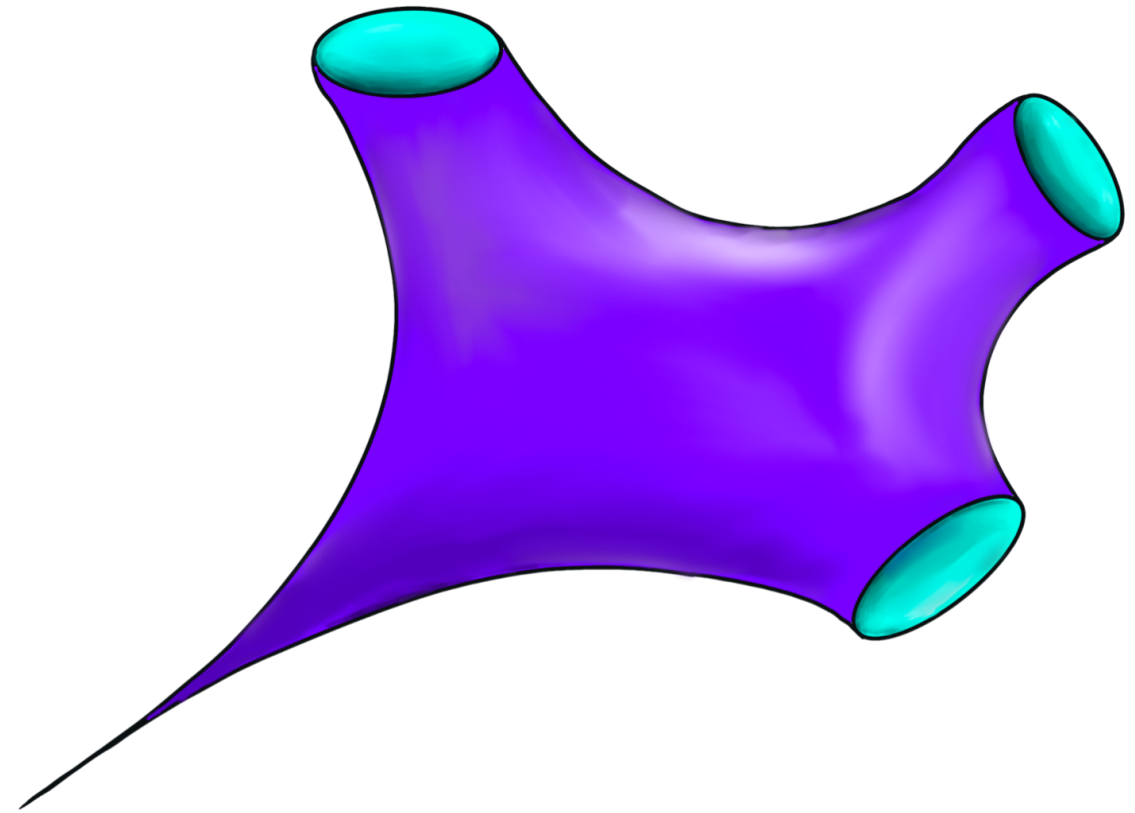


Twisting along self-intersecting curves

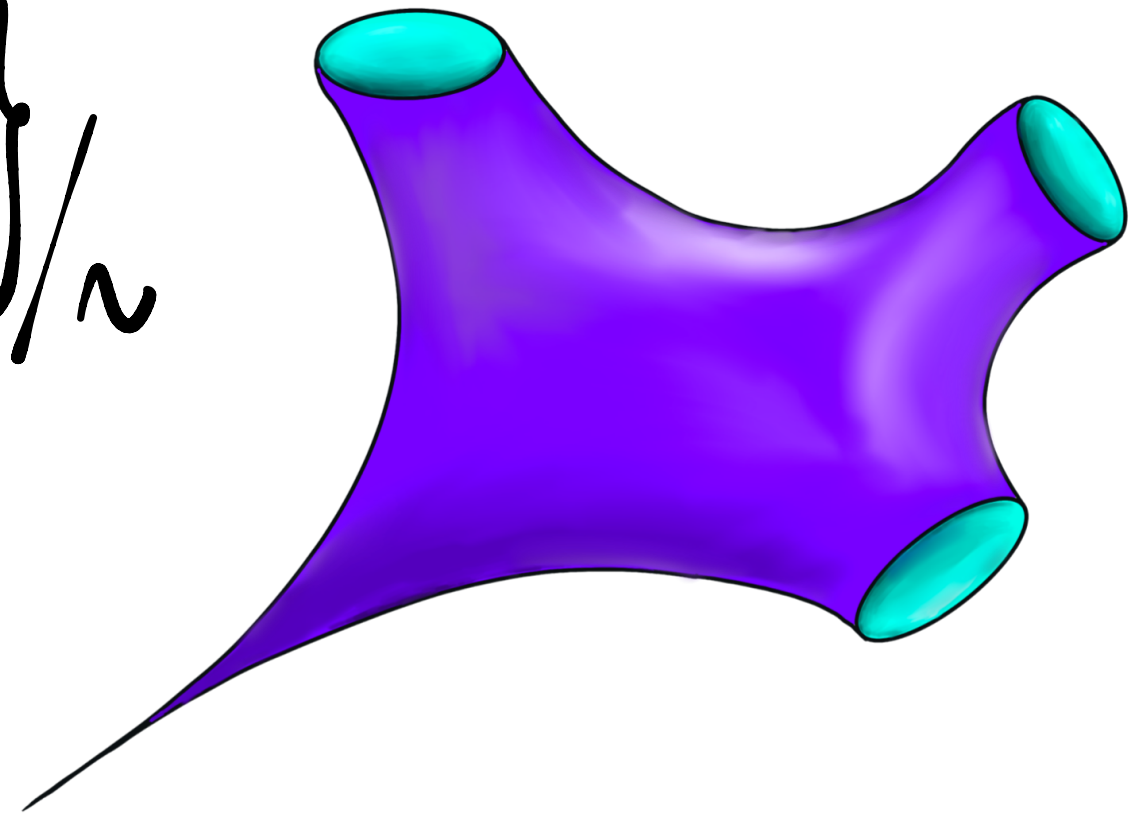
S

$=$



Twisting along self-intersecting curves

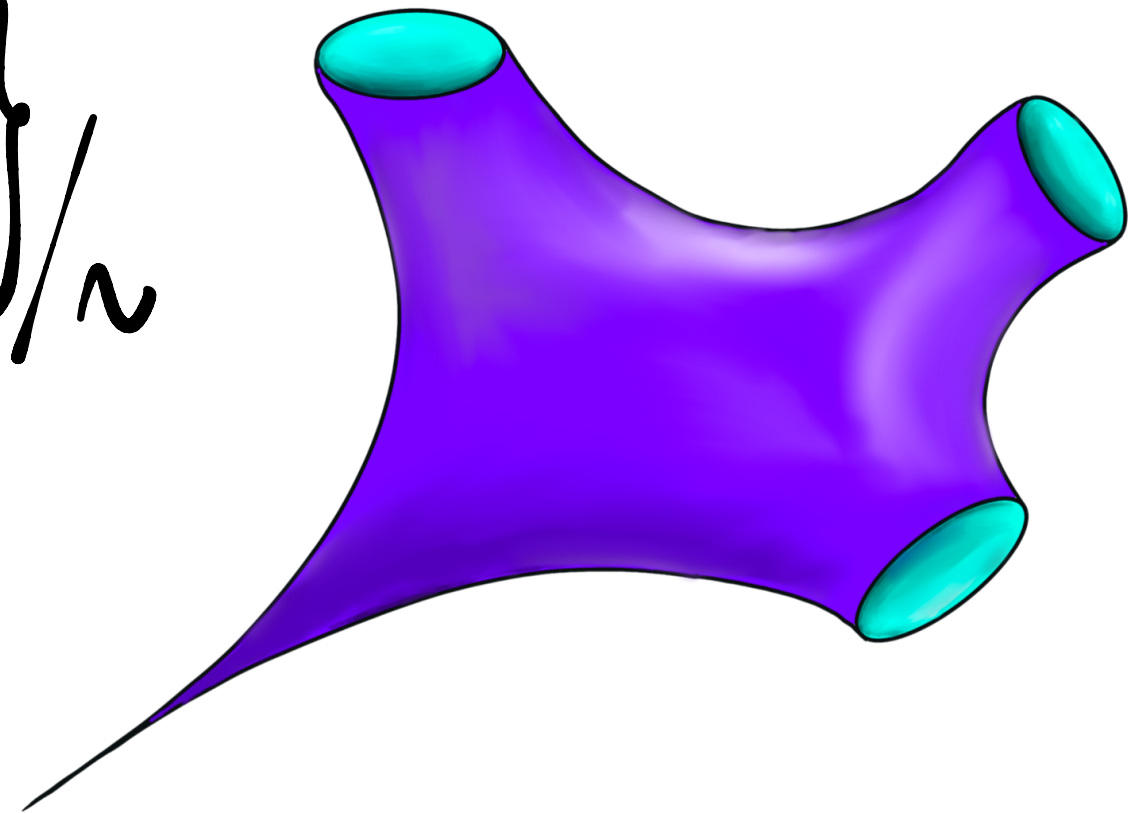
$$T(S) = \left\{ \begin{array}{c} \text{hyperbolic metrics} \\ \text{on } S \end{array} \right\} / \sim$$



Twisting along self-intersecting curves

$$T_L(S) = \left\{ \begin{array}{c} \text{hyperbolic metrics} \\ \text{on } S \end{array} \right\} / \sim$$

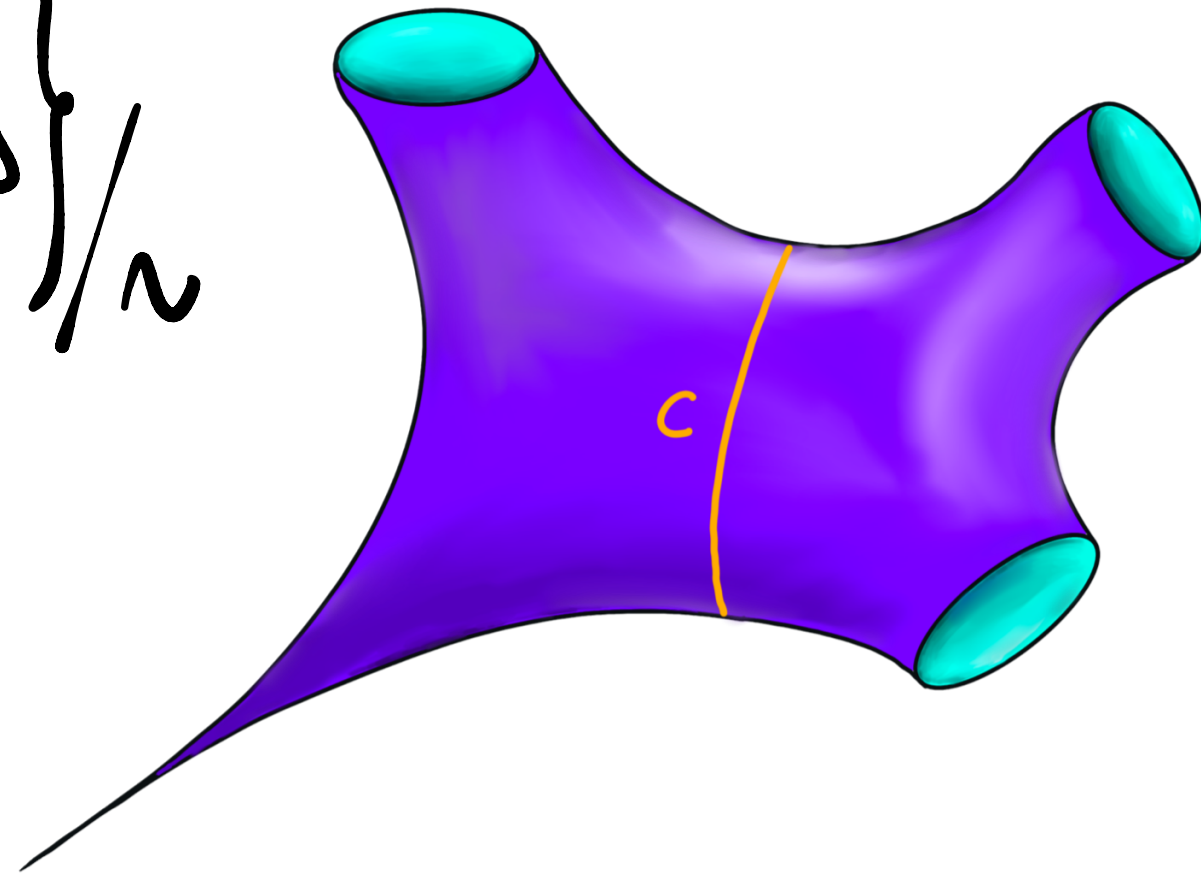
lengths of ∂



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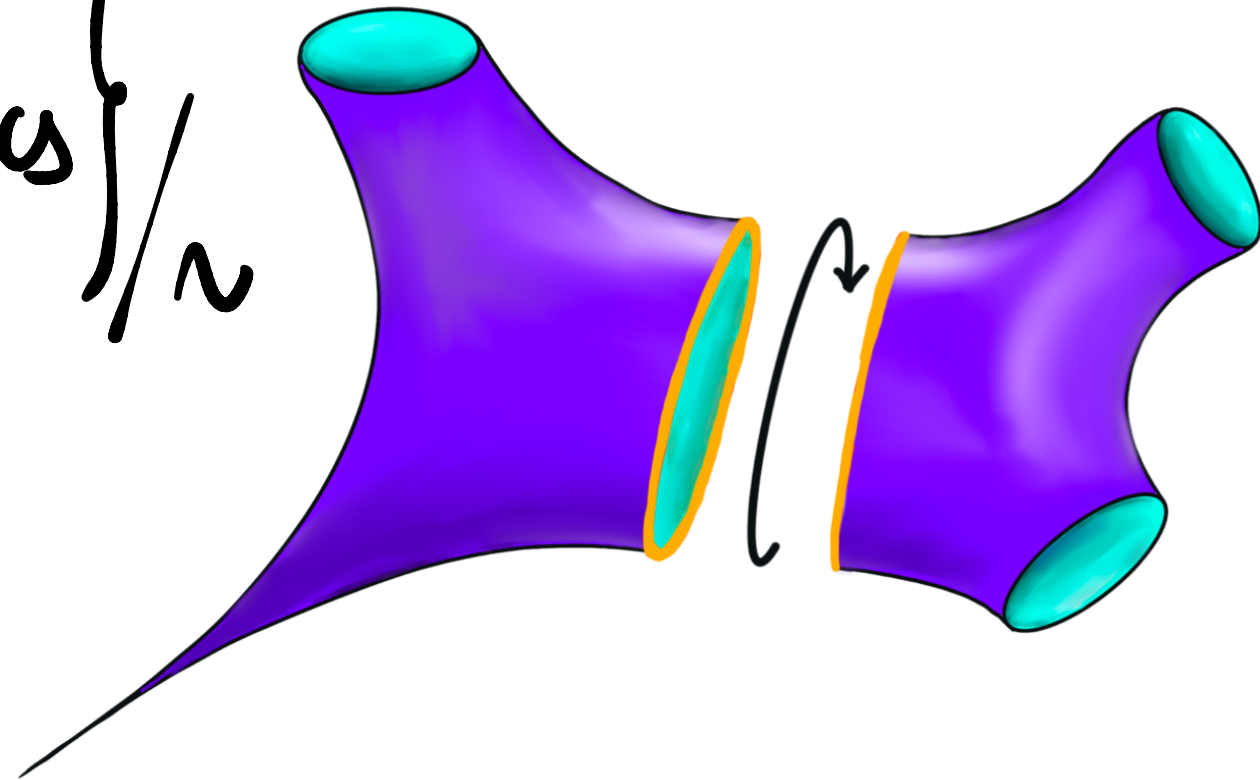
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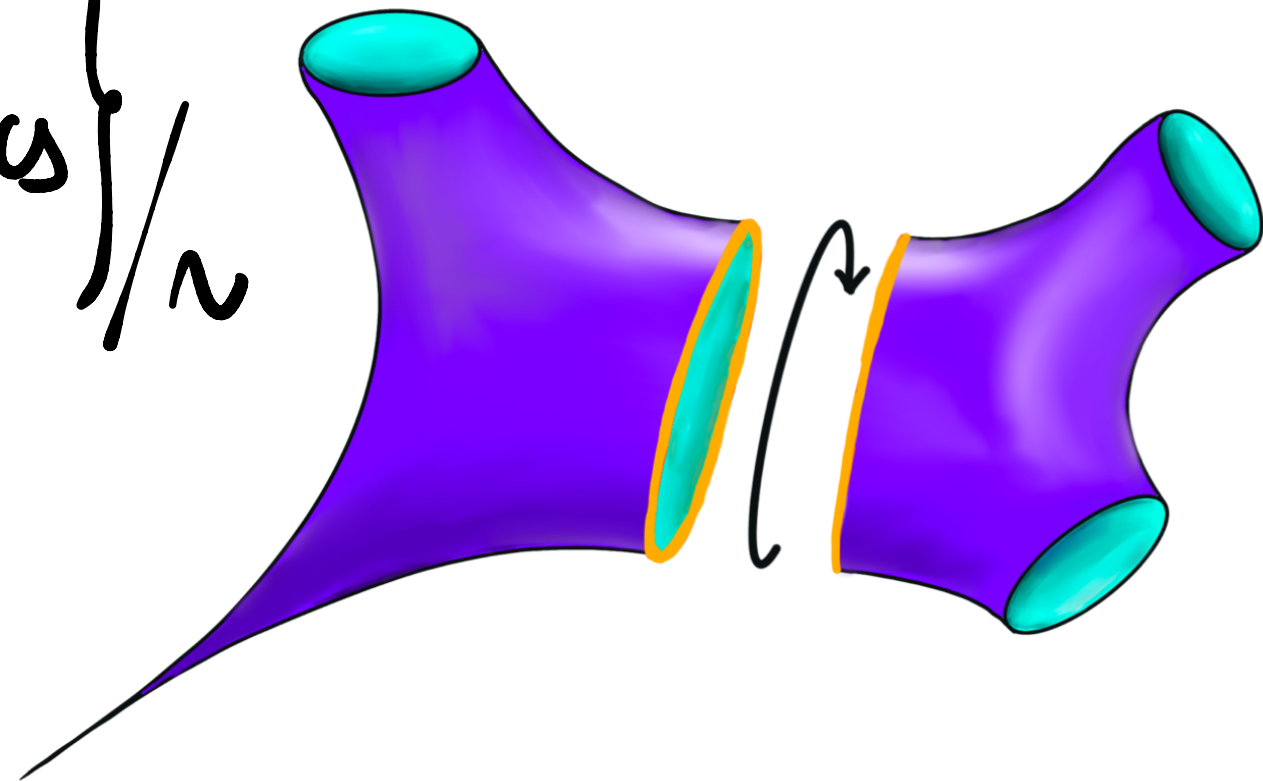
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lengths of ∂

Weil
-Peterson
)

ω_{WP}



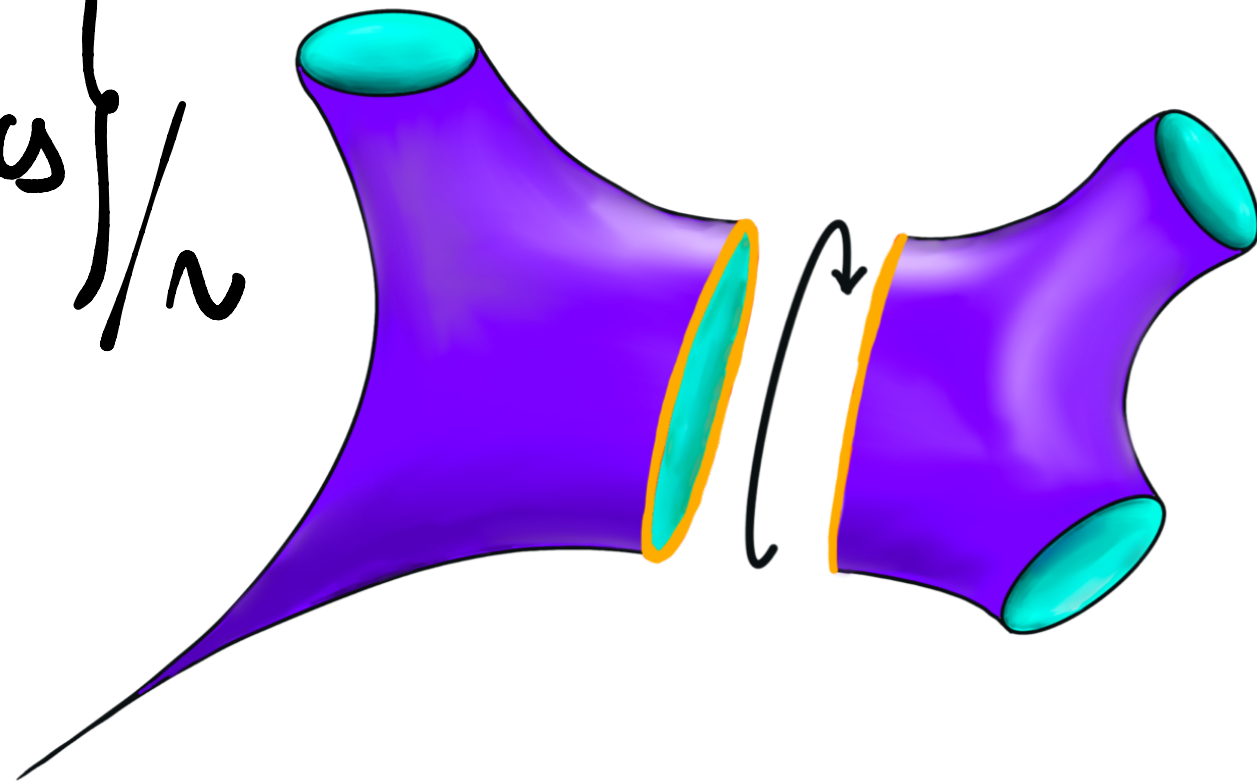
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Weil
-Peterson
)

ω_{WP}



l_c

length of simple curve

Twisting along self-intersecting curves

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lengths of ∂

Weil
-Peterson

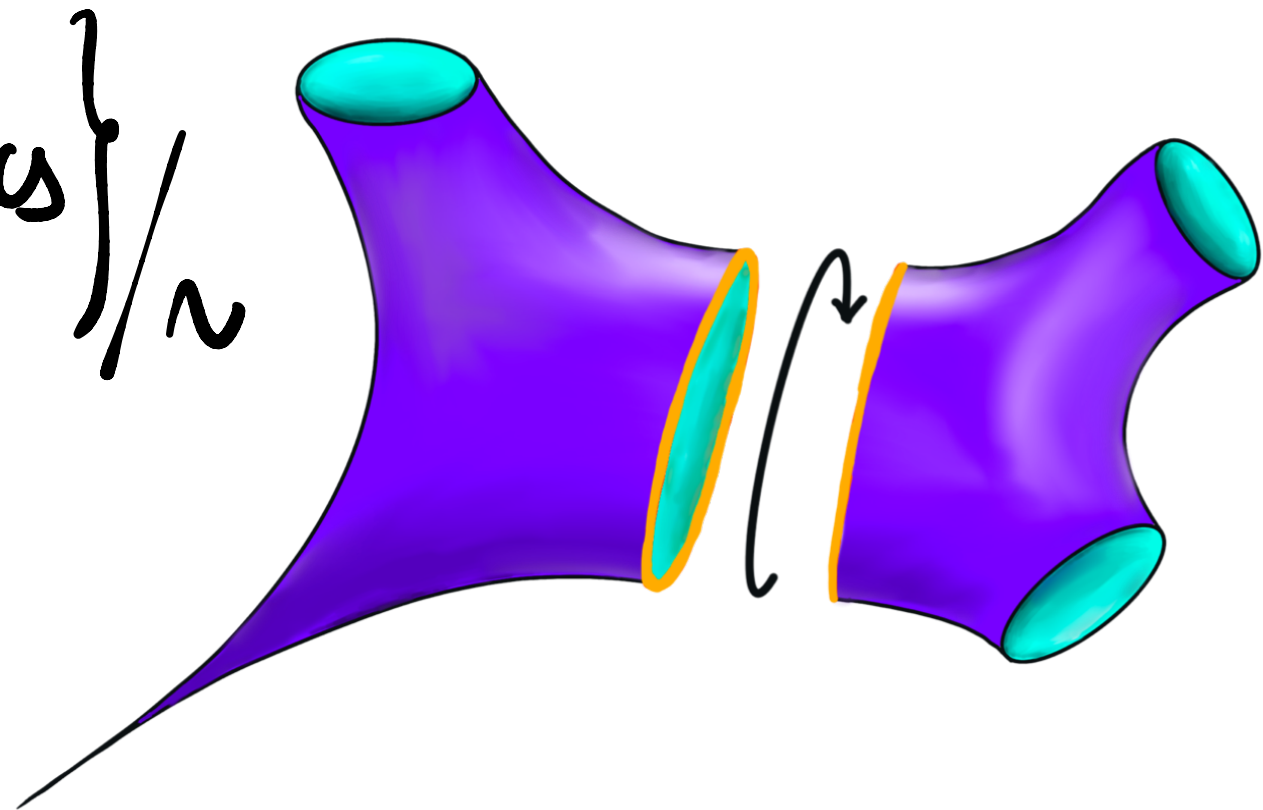
Hamiltonian
vector field

$$\omega_{WP}(Hl_c, \cdot)$$

$\parallel$

$$dl_c(\cdot)$$

length of simple curve



Twisting along self-intersecting curves

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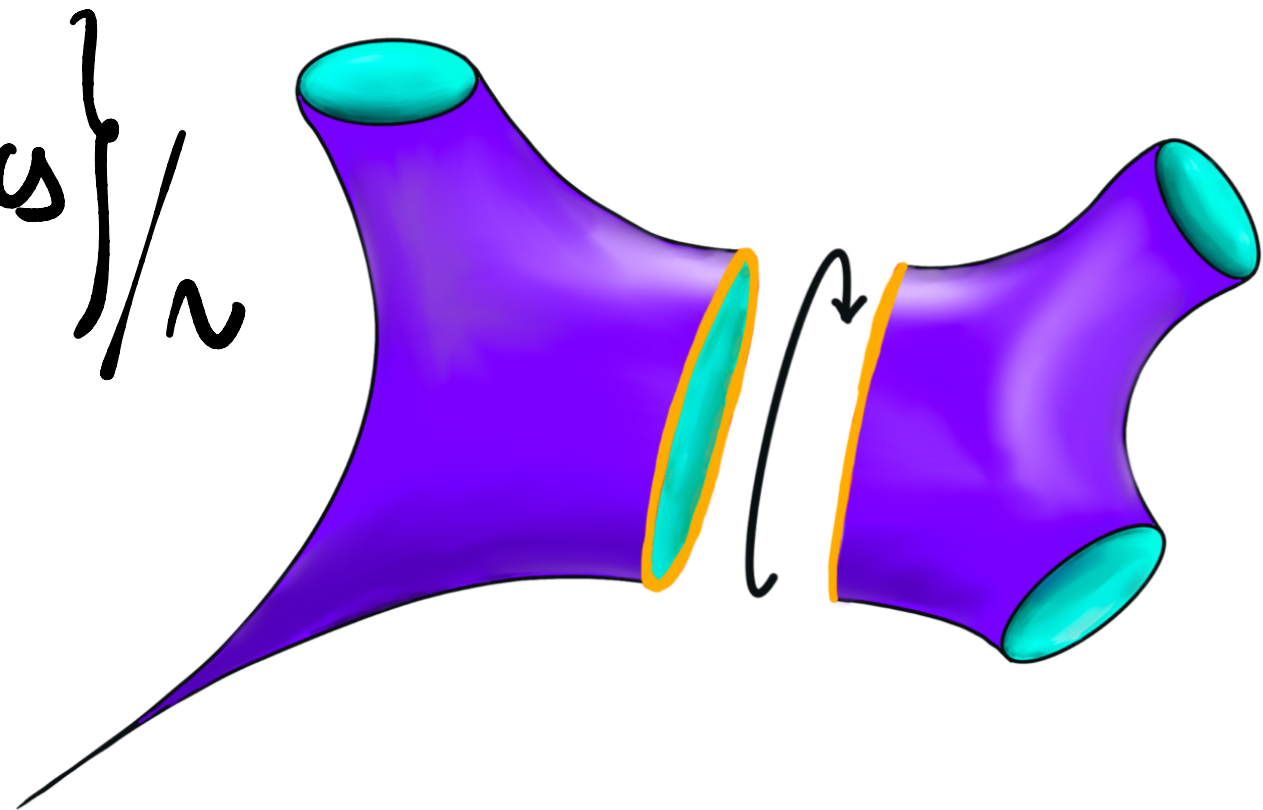
Hamiltonian
vector field

$$\omega_{WP}(tw_c, \cdot)$$

|| Wolpert '82

$$dl_c(\cdot)$$

length of simple curve



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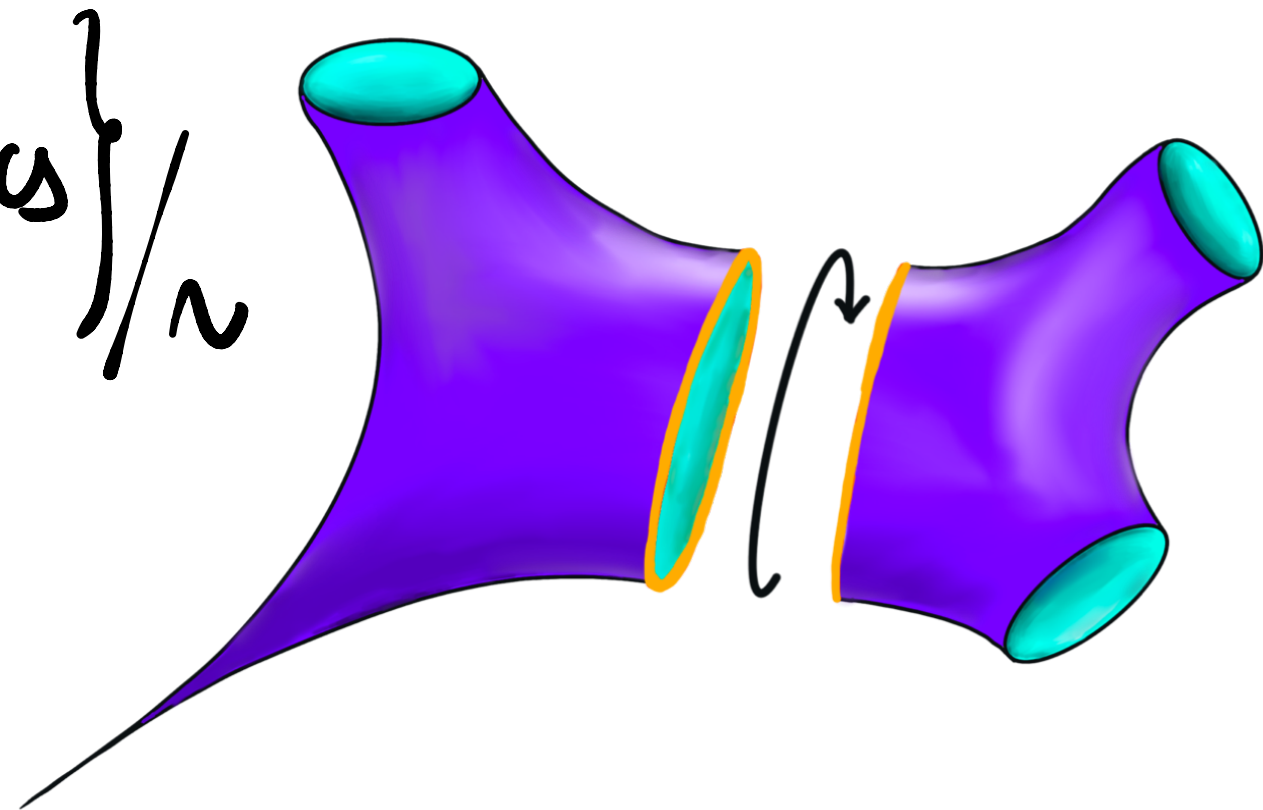
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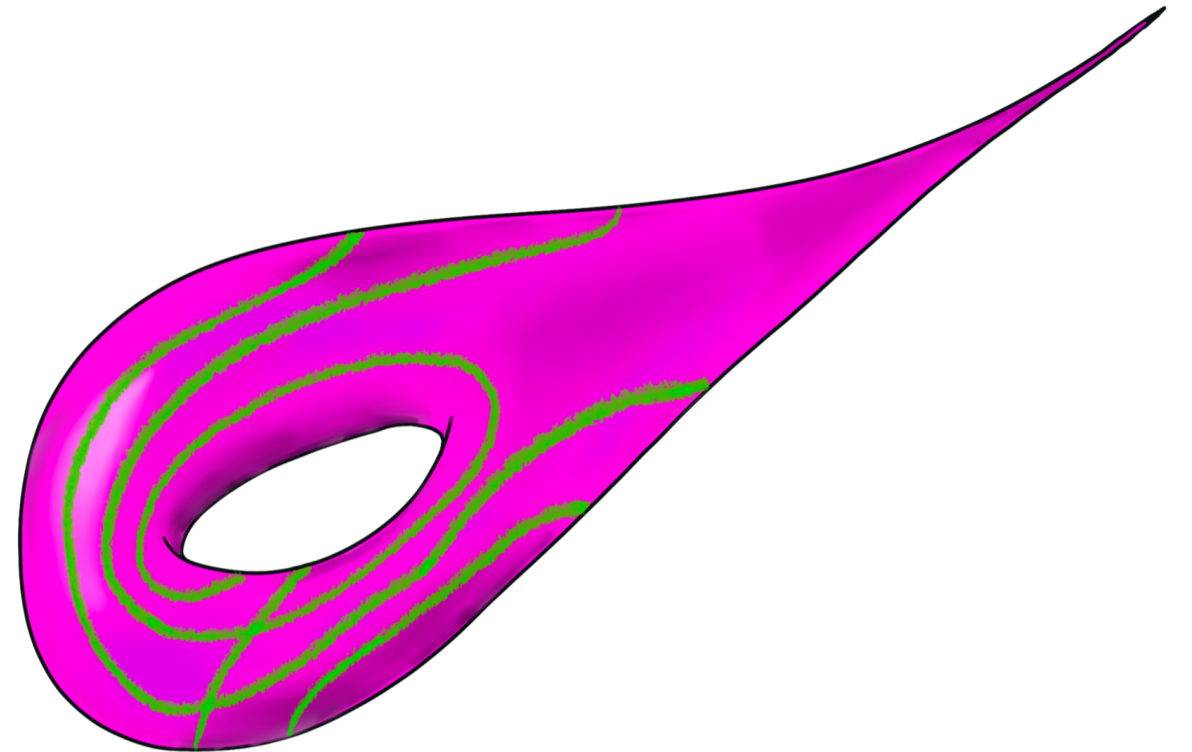
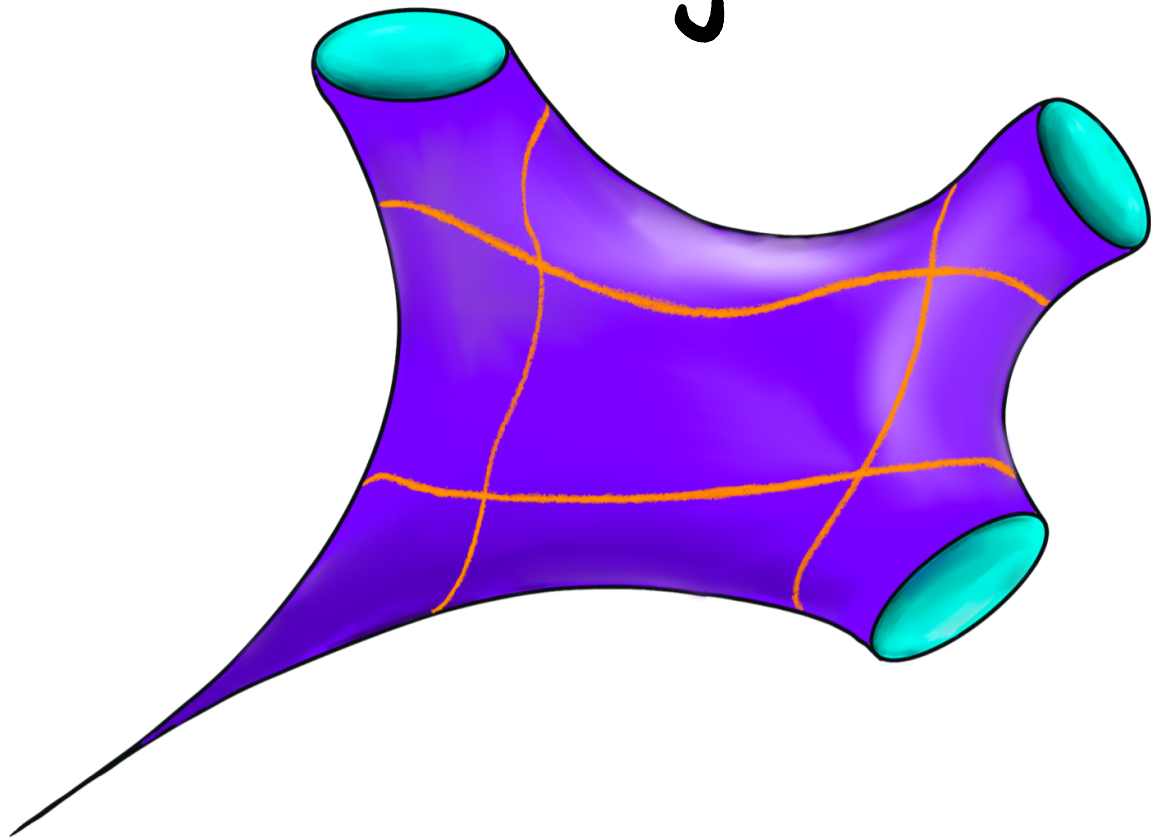
$$dl_c(\cdot)$$

length of simple curve

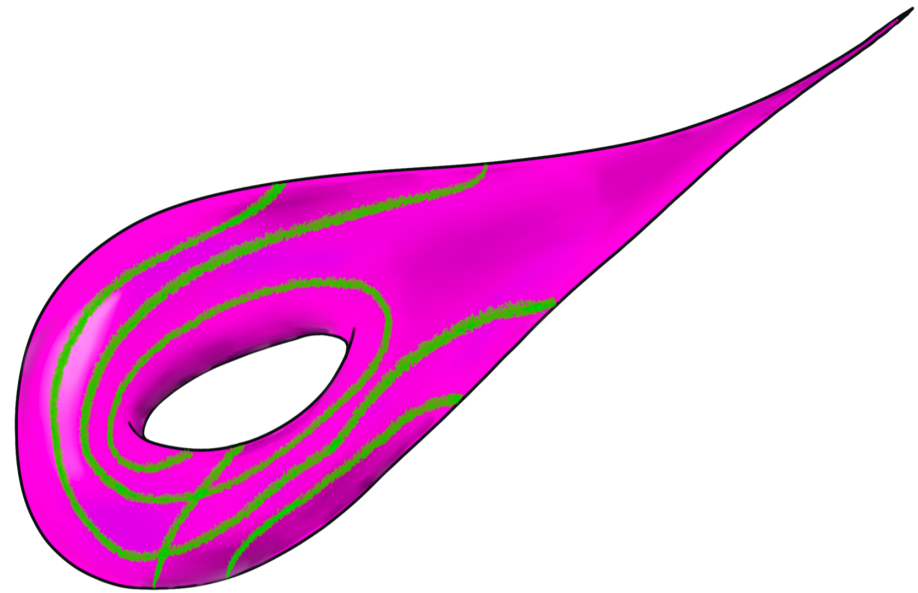
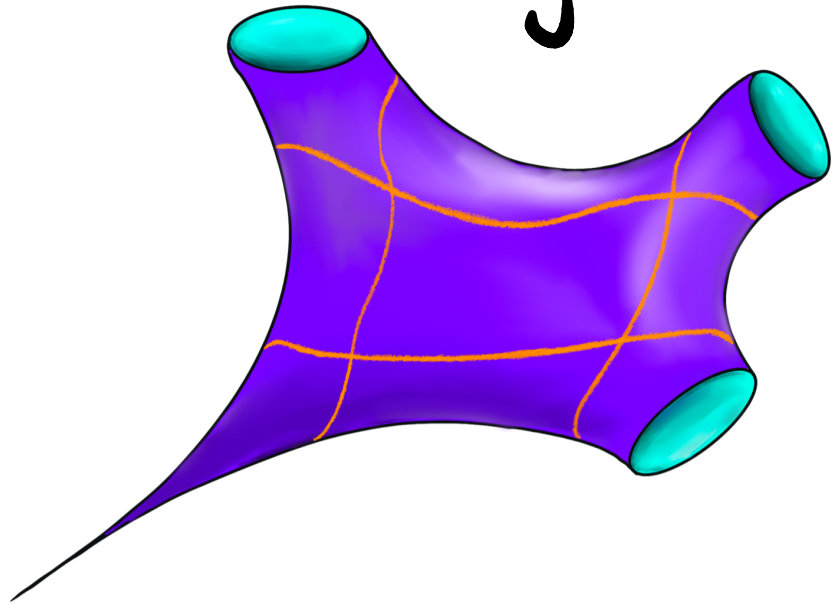


Q: What about
self-intersecting
curves?

Thm [CC'25] Let c be a filling curve
in one of



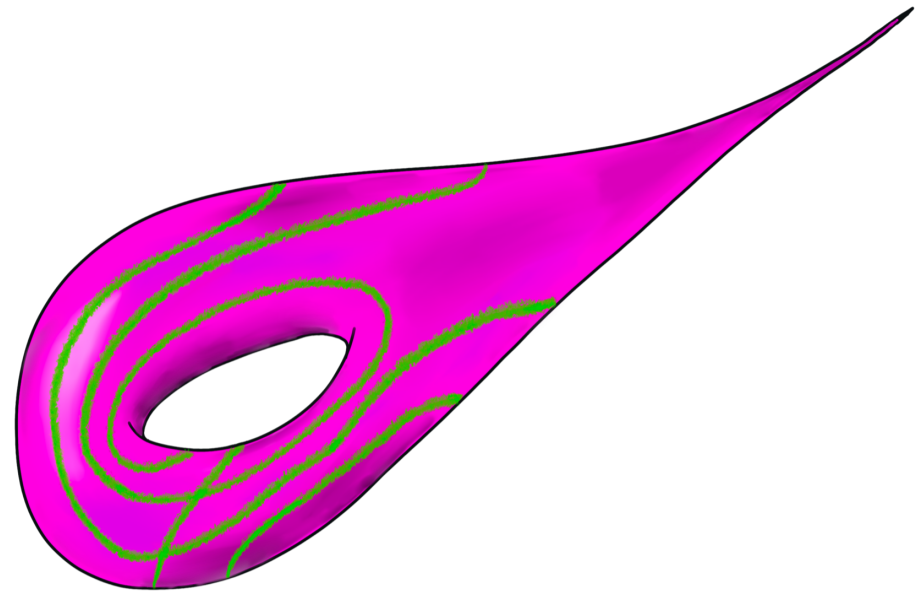
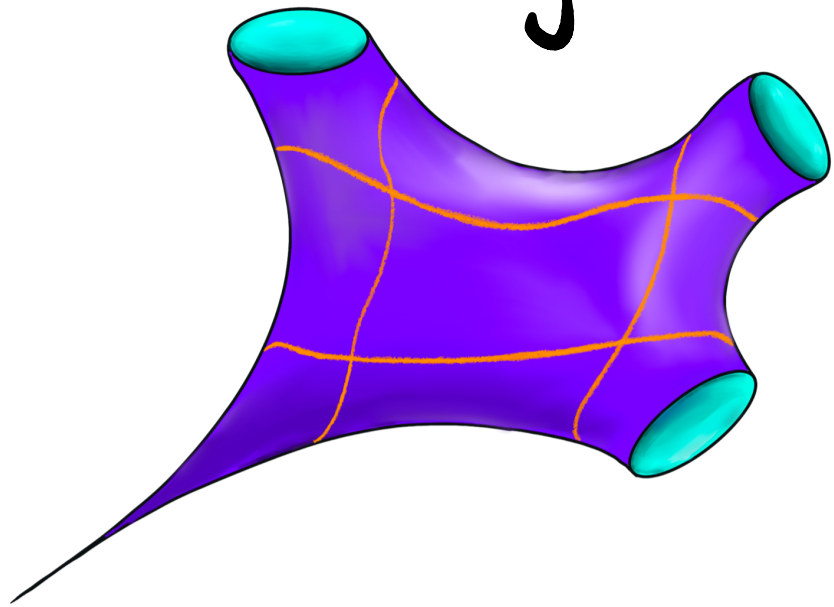
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Then every orbit of the Hamiltonian flow of
 $\mathcal{L}_c: T_L(S) \rightarrow \mathbb{R}$ is periodic.

Thm [CC'25] Let c be a filling curve

in one of



Then every orbit of the Hamiltonian flow of $L_c: T_L(S) \rightarrow \mathbb{R}$ is periodic.

→ follows directly from Thurston's earthquake thm & Kerckhoff's convexity thm.

Going to higher rank:

$$\mathcal{L}(S) = \left\{ \begin{array}{l} \text{real convex} \\ \text{projective str.} \end{array} \right\} / \sim$$

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$$\mathcal{L}(S) = \left\{ \begin{array}{l} \text{real convex} \\ \text{projective str.} \end{array} \right\} / \sim \subseteq \text{Hom}(\pi_1 S, \text{SL}_3 \mathbb{R}) / \sim$$

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length
data

Atiyah-
Bott - ω
Goldman

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length
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$$[p] \mapsto \text{tr}(p(c))$$

$$\ddot{\text{tr}}_c$$

Atiyah-
Bott-
Goldman ω

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Atiyah-
Bott-
Goldman

$$\omega(H\text{tr}_c, \cdot) = d\ddot{\text{tr}}_c(\cdot)$$

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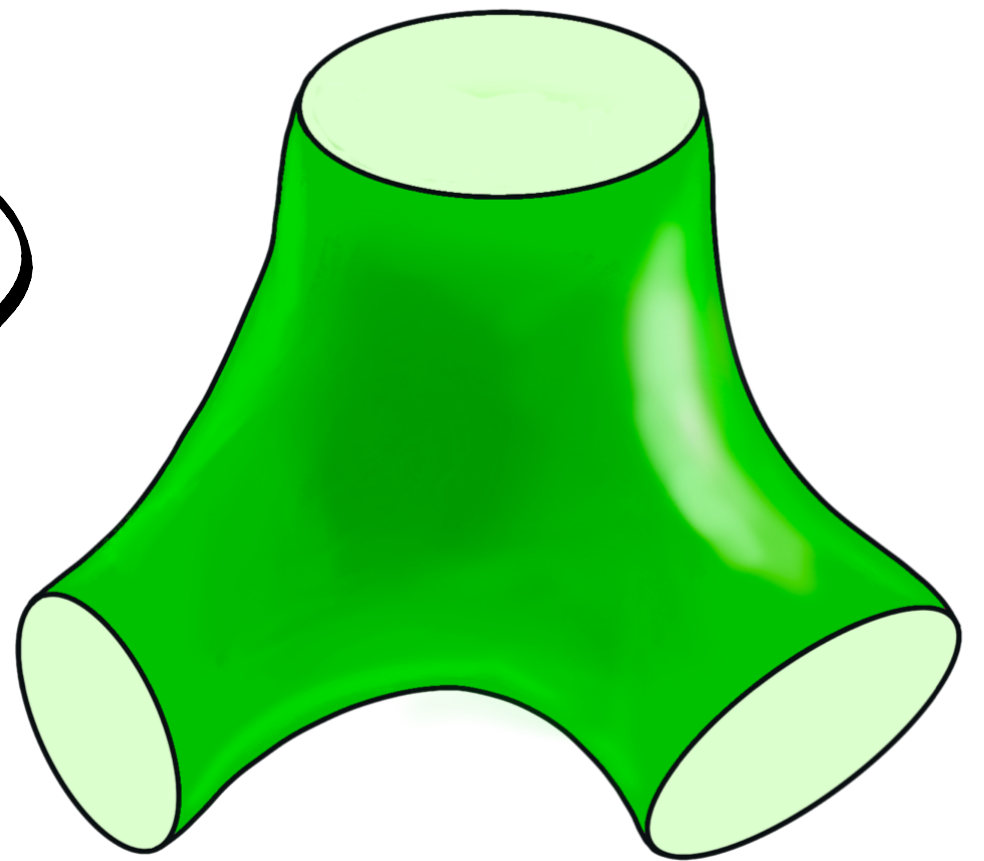
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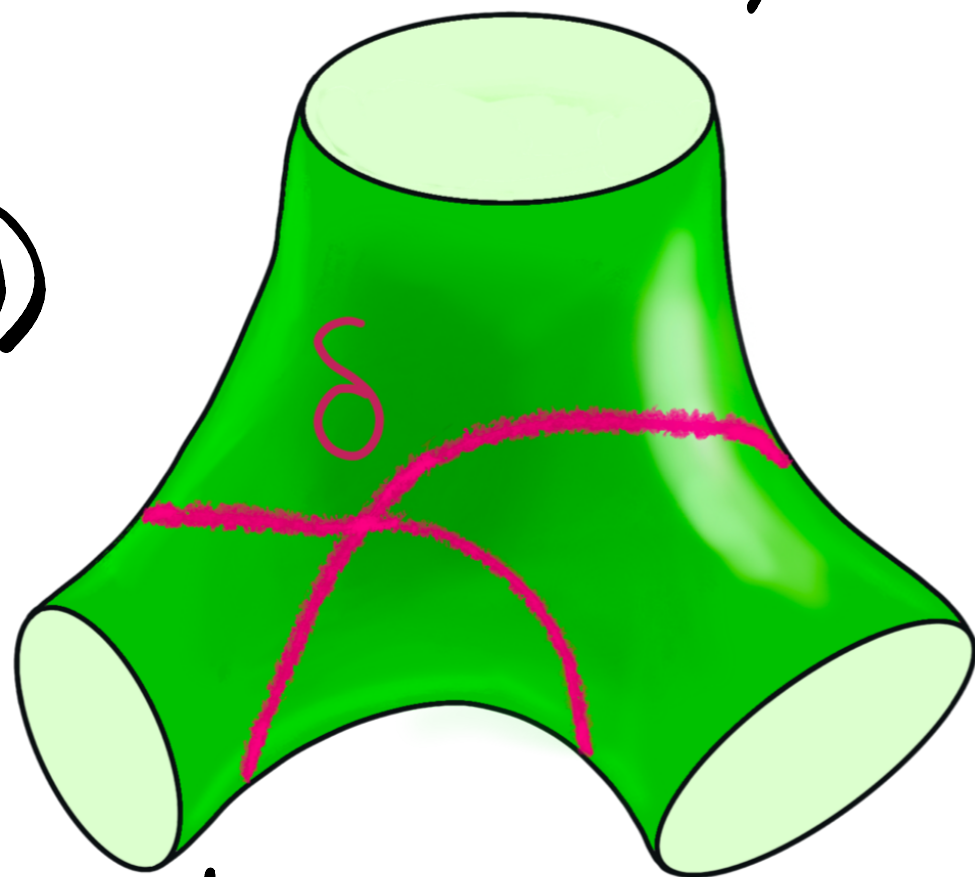
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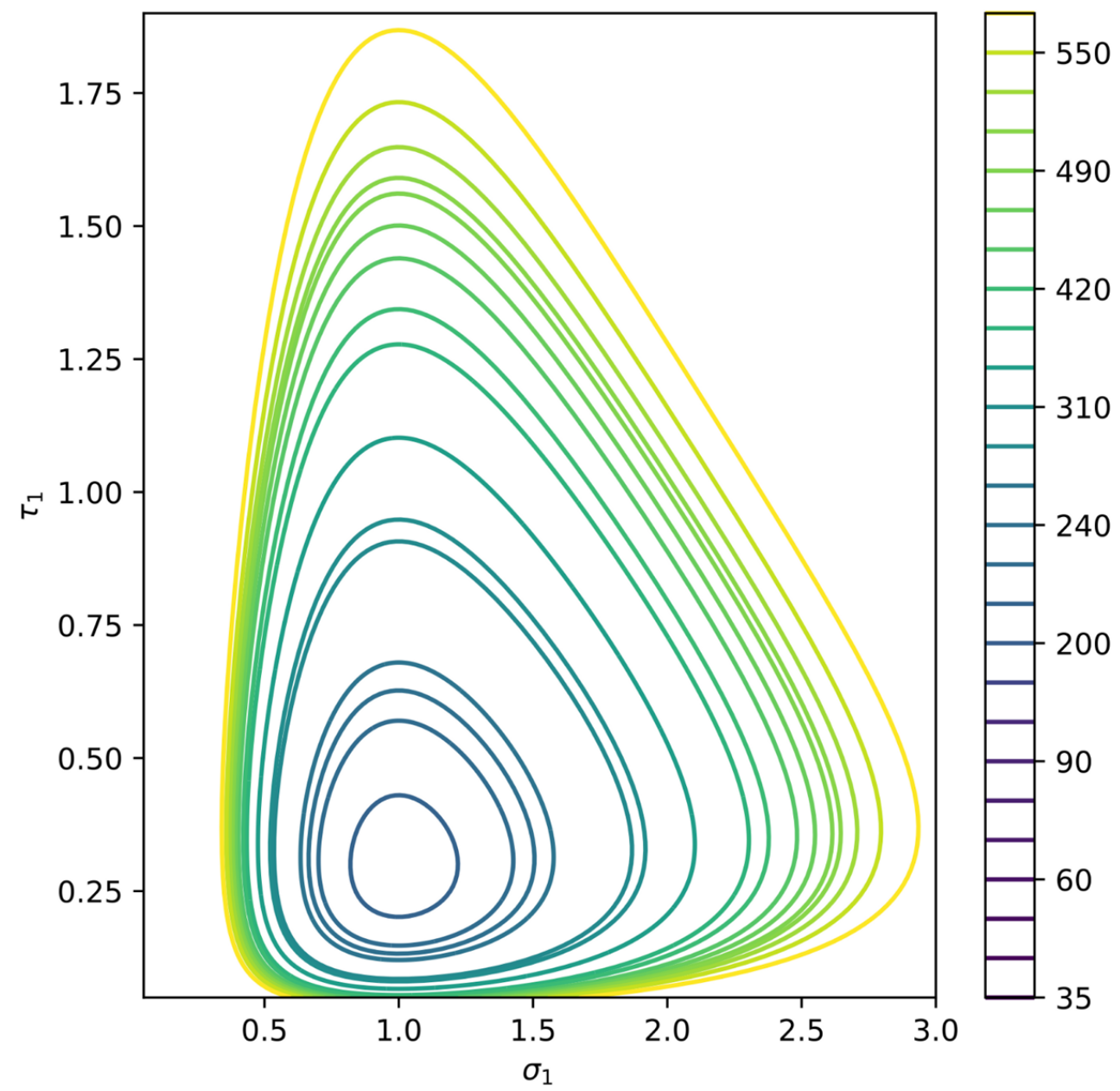
Atiyah-
Bott-
Goldman

$$\omega(\text{Htr}_c, \cdot) = d \text{tr}_c(\cdot)$$



Thm [CC'25]: Every orbit of the

Hamiltonian flow of $\text{tr}_\delta: \mathcal{L}_L(\text{hourglass}) \rightarrow \mathbb{R}$ is periodic.



Thank you!

