

Improving Acylindrical Actions on Trees

Higher Dimensional Hyperbolic Geometry

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Acylindrical Actions on Trees

Picture

$\geq k$

u v

$\Rightarrow |Stab| \leq C$

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- ▶ Quasi-convex splittings (Gitik–Mj–Rips–Sageev 1998), e.g. JSJ-decompositions of hyperbolic groups.
- ▶ Fundamental groups of **non-geometric** closed and orientable 3-manifolds (Wilton–Zaleskii 2013).

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- ▶ **We therefore want control over vertex stabilisers!**

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- ▶ G admits a non-elementary acylindrical splitting with finitely generated finite height edge stabilisers.

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Let H be a finitely generated subgroup of a one-relator group with an acylindrical Magnus hierarchy. If we assume this conjecture then H is virtually compact special.