

# Generalised triangle groups

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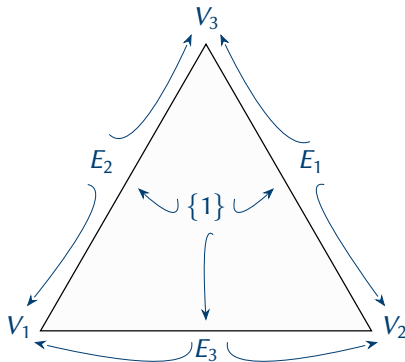
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A *generalised triangle group* is a **triangle of groups** with trivial face group

# Structure theorem

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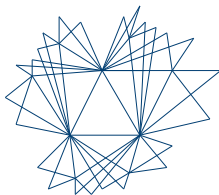


Figure: A part of the complex  $X$  for a trivalent triangle group

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  - ▶ Candidates for non-residually finite hyperbolic groups

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## Incoherence

- ▶ Conjecturally **all incoherent** (valence  $\geq 3$ )
- ▶ Main tool: **fibring** (for eligible examples)
  - ▶ Bestvina-Brady Morse theory
  - ▶ Kielak's approach

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## Automaticity

[1, 25+] k-fold triangle groups are automatic.

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Thank you.