Dehn functions of Bestvina-Brady groups

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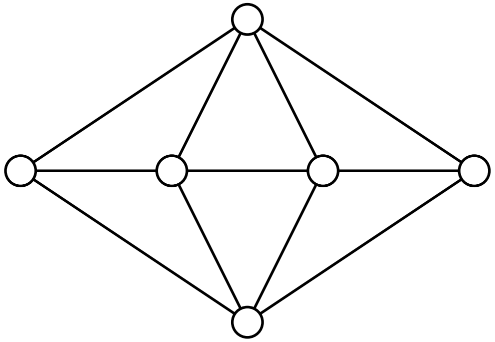
Well-known: $d_\Gamma = 1 \Leftrightarrow \mathrm{BB}_\Gamma$ hyperbolic $\Leftrightarrow \Gamma$ is a tree.

Examples



$$d_\Gamma = 3$$

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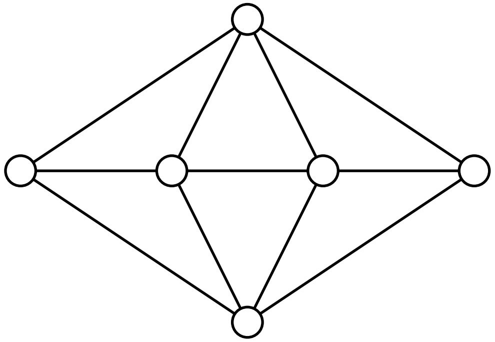

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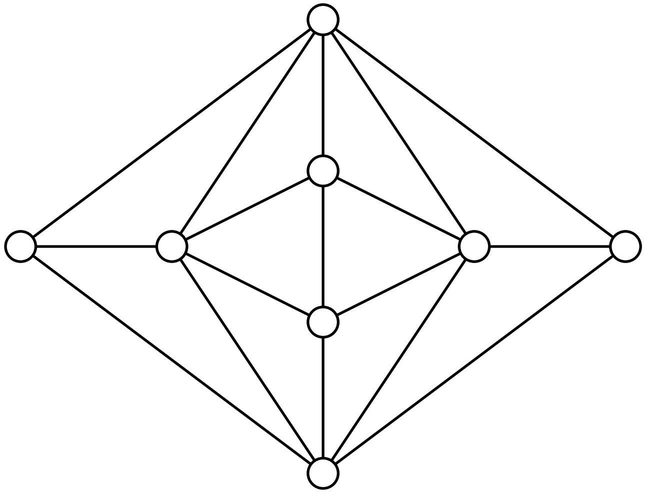

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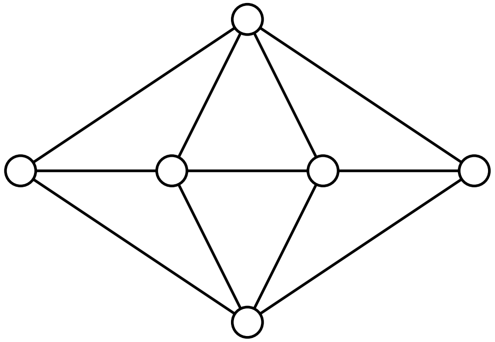

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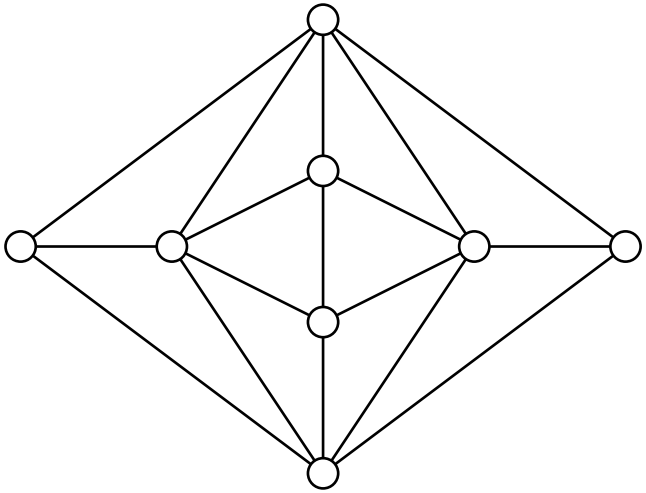

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Idea: look at Maximal Reducible Subgraphs.

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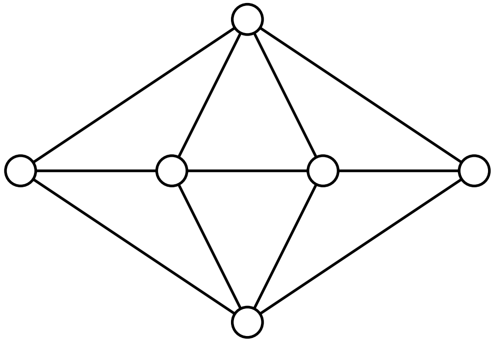

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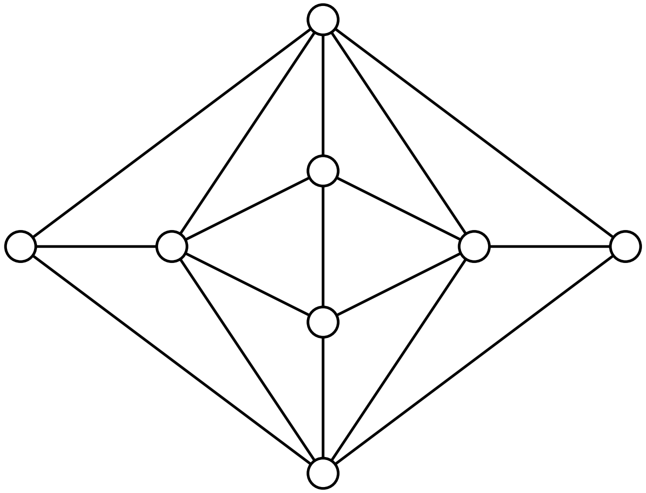

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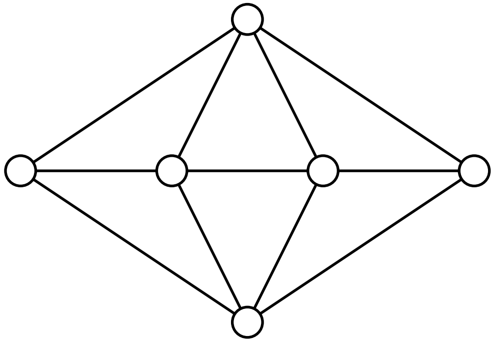

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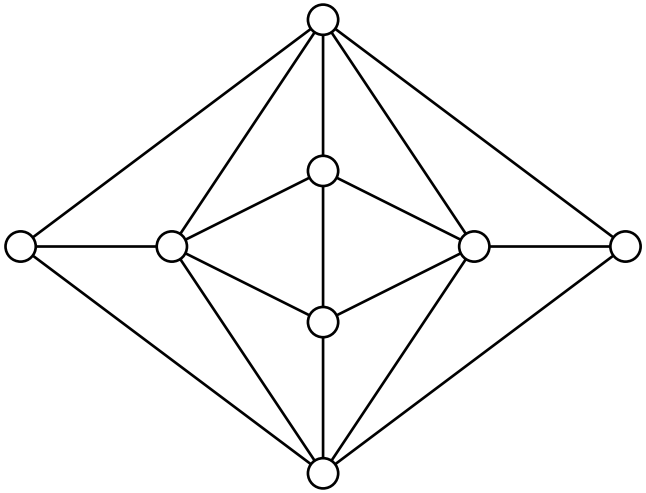

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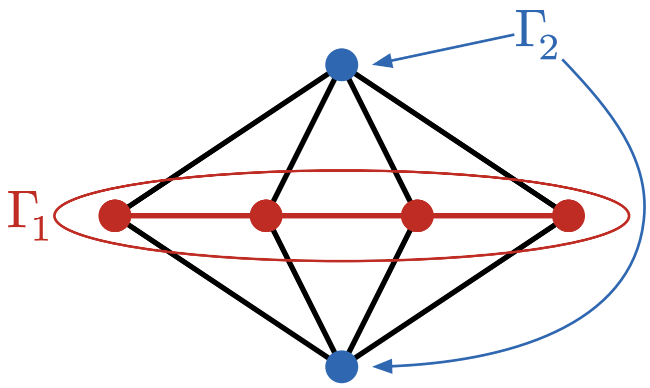

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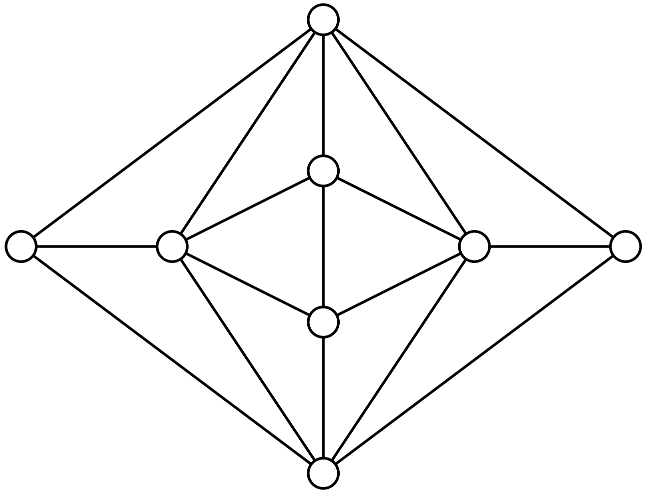

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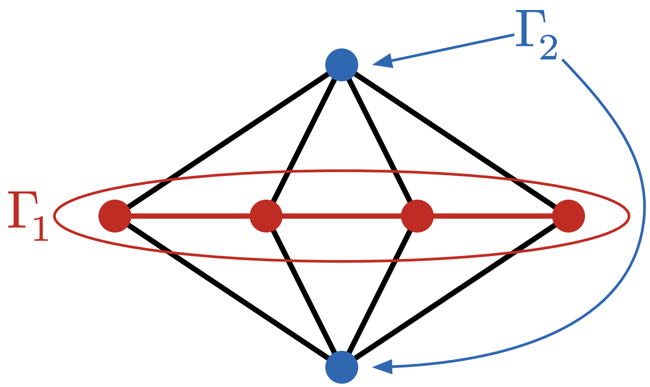

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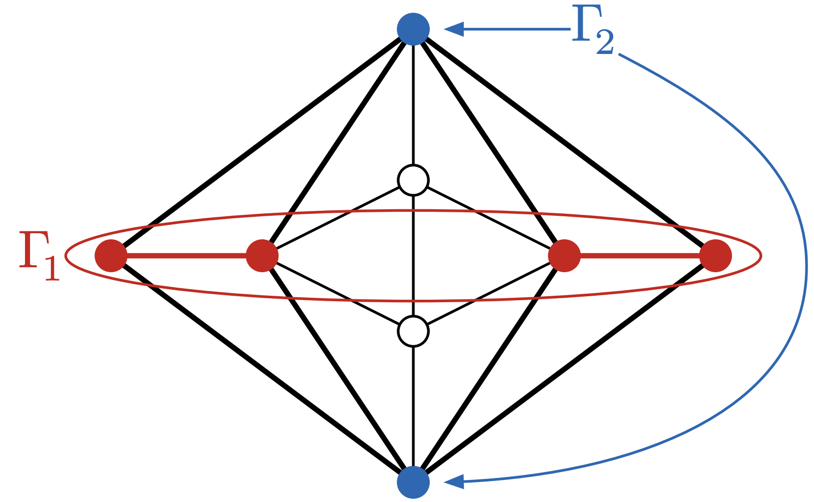

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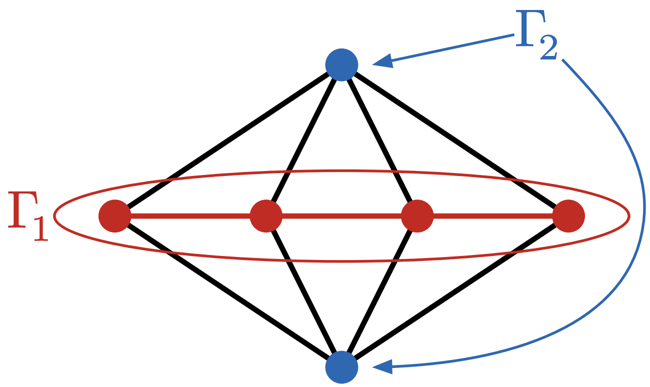

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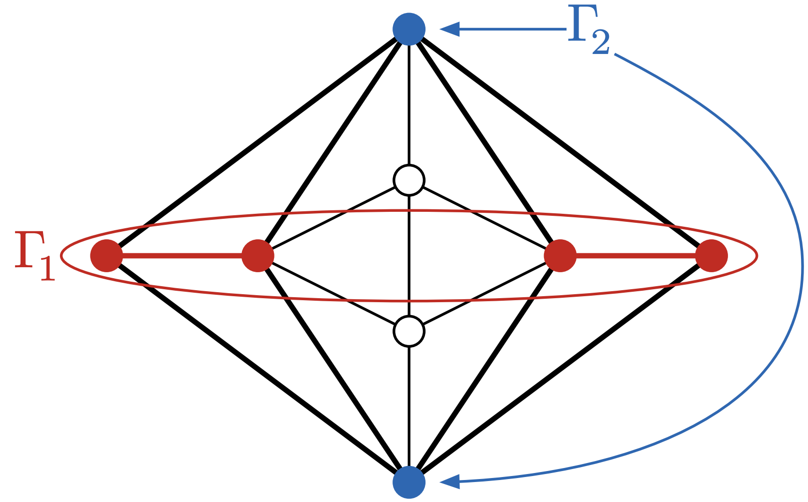

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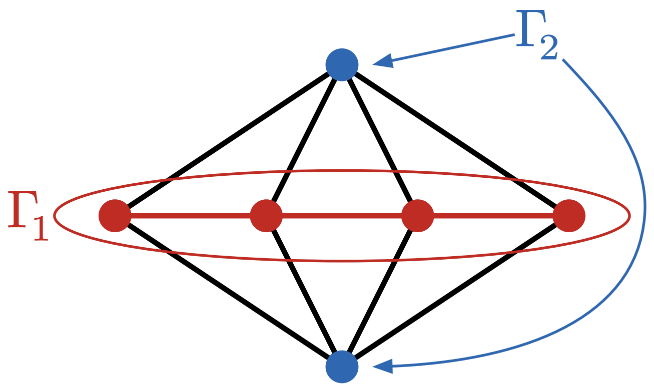

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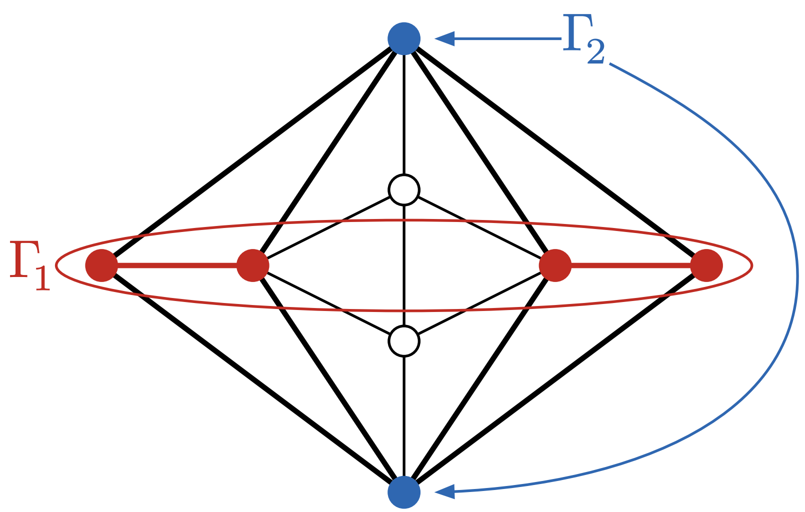

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- $d_\Gamma \geq 3 \Leftrightarrow \Gamma$ contains a MRS that is a join of two *irreducible graphs*, each with at least two vertices.
- $d_\Gamma = 4 \Leftrightarrow \Gamma$ contains a MRS that is a join of two *disconnected graphs*.

Thank you!