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# Effective count of 3×3 integral matrices with a specified characteristic polynomial

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# Setup

- Let $p \in \mathbb{Z}[\lambda]$ be an integral characteristic polynomial of degree $3$ which splits over $\mathbb{R}$ but irreducible over $\mathbb{Q}$.
- It cuts out a variety $\mathscr{V}_{3,p}$ in the vector space of $3 \times 3$ real matrices.
- Then, $\mathrm{SL}_3(\mathbb{R})$ acts on the variety $\mathscr{V}_{3,p}$ by conjugation.

# Effective count of 3×3 integral matrices

- Let $\mathcal{N}_{3,p}(T) = \#\{L \in \mathscr{V}_{3,p} : \|L\| \leq T\}$ using the Frobenius norm.

**Theorem (S. '25)**

$\exists\, c_p > 0, \kappa > 0$ *such that*

$$\mathcal{N}_{3,p}(T) = c_p T^3 + O_p(T^{3-\kappa}) \qquad \forall\, T > 0.$$

- The main term is due to Eskin–Mozes–Shah '96.

# Effective equidistribution of translates of tori

- Let $A = \begin{pmatrix} * & & \\ & * & \\ & & * \end{pmatrix}$ and $N_\epsilon = \left\{ \exp\begin{pmatrix} 0 & x & z \\ & 0 & y \\ & & 0 \end{pmatrix} : |x|, |y| \geq \epsilon\sqrt{x^2+y^2+z^2} \right\}$ for any $\epsilon > 0$.

**Theorem (S. '25)**

*Let $\Gamma < \mathrm{SL}_3(\mathbb{R})$ be an arithmetic lattice. There exists $\kappa > 0$ and $\Lambda > 0$ depending only on $\Gamma$ such that the following holds.*

*Let $x_0 \in \Gamma\backslash\mathrm{SL}_3(\mathbb{R})$ such that $Ax_0$ is periodic. Let $n \in N_\epsilon$ for some $\epsilon > 0$ and $T := \|\log(n)\| \gg_{\Gamma,\mathrm{ht}(Ax_0)} \epsilon^{-\Lambda}$. Then, for all $\phi \in C_c^\infty(\Gamma\backslash\mathrm{SL}_3(\mathbb{R}))$, we have*

$$\left| \int_{Ax_0} \phi(nx)\,\mathrm{d}\mu_{Ax_0}(x) - \int_{\Gamma\backslash\mathrm{SL}_3(\mathbb{R})} \phi\,\mathrm{d}\mu_{\Gamma\backslash\mathrm{SL}_3(\mathbb{R})} \right| \leq \mathcal{S}(\phi)\epsilon^{-\Lambda}T^{-\kappa}.$$

- The main term is due to Eskin–Mozes–Shah '96.

# Further questions

- Analogous theorems also hold for:

$$\mathrm{SO}(n,1)^\circ, \qquad \mathrm{SL}_2(\mathbb{R}) \times \mathrm{SL}_2(\mathbb{R}), \qquad \mathrm{SU}(2,1), \qquad \mathrm{Sp}_4(\mathbb{R}).$$

  What happens with other semisimple Lie groups with larger root systems?

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Thank you!