# Finding Clusters of Primes, I

## Progress Report 2003 - 2005

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## 1 Purpose

In the current application of our parallelization concept we are using an algorithm involving sieving techniques for locating and counting clusters of prime numbers. Whereas the distribution of primes seems to be fairly regular (if the Riemann hypothesis is true), the distribution of twin primes and longer clusters is largely unknown and is characterized by large-scale anomalies. Collecting experimental data on these anomalies is one of the reasons for the interest in clusters of primes.

Another challenge of finding clusters of primes is collecting data supporting the unproven *prime k-tuple hypothesis*, which is concerned with sequences

$$\mathbf{c} = [c_1, c_2, \ldots, c_k]\,, \quad c_j \in \mathbb{Z}\,, \quad j = 1, 2, \ldots, k\,, \quad c_1 < c_2 < \cdots < c_k$$

of $k$ integers. The sequence or *pattern* $\mathbf{c}$ is called *admissible* if its elements leave at least one residue class empty modulo every prime. As a consequence of this definition, simple divisibility considerations cannot exclude the existence of integers $x$ such that each element of the shifted pattern $\mathbf{c}_x = [x + c_1, x + c_2, \ldots, x + c_k]$ is prime. A shifted pattern $\mathbf{c}_x$ with this property will be referred to as a *prime instance* of $\mathbf{c}$. The prime k-tuple hypothesis states that every admissible pattern has *infinitely many* prime instances.

An example of a non-admissible pattern is $\mathbf{c} = [0, 2, 4]$; modulo 3 all three residue classes are occupied. For every integer $x$ the shifted pattern $\mathbf{c_x} = [x, x + 2, x + 4]$ contains one multiple of 3; therefore there exist only finitely many prime instances, namely for $x = 3$ and $x = -7$, when one of the elements is the prime 3 itself or its opposite.

The goal of this project is to find and display prime instances of certain admissible patterns, in particular of the densest possible patterns for a given number $n$ of elements. With the current software and hardware $n \leq 21$ and $|x| \leq 10^{30}$ delimit the region of feasible search.

Many dense admissible patterns may be defined by a finite subsequence of consecutive positve or negative odd primes. E.g. $\mathbf{c} = [17, 19, 23, 29, 31]$ is admissible, it reoccurs as the prime instance $\mathbf{c}_{210} = [227, 229, 233, 239, 241]$, and many more may be found. In general, the inverse (i.e.the mirror image) of an admissible pattern is again admissible: $\mathbf{c}' = [-31, -29, -23, -19, -17]$ has the prime instance $\mathbf{c'}_{1320} = [1289, 1291, 1297, 1301, 1303]$, etc.

In this text, admissible patterns will be denoted by listing the elements $c_j$ in increasing order, separated by commas and enclosed in square brackets (often normalized such that $c_1 = 0$). For brevity we will abbreviate a list of consecutive primes by merely indicating the bounding elements, seprated by two periods: $\mathbf{c} = [17..31]$, $\mathbf{c}' = [-31.. - 17]$.

A proof of the prime k-tuple hypothesis is currently out of reach; not even for the simplest case, the twin prime hypothesis, a proof is in sight. In contrast, the observed average densities of prime k-tuples in the accessible range are in perfect agreement with the densities $\rho_{\mathbf{c}}(x)$ conjectured by Hardy and Littlewood [5] in 1922:

$$\rho_{\mathbf{c}}(x) = \frac{h_{\mathbf{c}}}{(\log x)^{|\mathbf{c}|}}, \tag{1}$$

where $h_{\mathbf{c}}$ is the Hardy-Littlewood constant associated with the pattern $\mathbf{c}$, and $|\mathbf{c}|$ is the number of elements in the pattern $\mathbf{c}$.

# 2 Review of Results 2001 - 2003

The main achievement in the preceding period was the discovery of two dense clusters of 18 primes in the range of $3 \cdot 10^{24}$ on November 13, 2000 and on January 31, 2001. This computation was about 50 times harder than finding the 23-digit clusters of 17 primes among 67 consecutive integers

(first discovered by Tony Forbes [3], and indepently by J. Waldvogel [10], both in 1998). The news about this discovery was immediately announced in the number theory press [6]. It also got coverage in the web journal of ETH (ETH Life, December 6, 2000, [2]). Dense clusters of primes receive particular attention on the website [4], continuously actualized by Tony Forbes. The successes of our implementation bear the danger of monopolizing this site and taking away all the fun!

We approched our "prime" target, the search for clusters of 18 primes among 71 consecutive integers, by searching blocks of size $10^{24}$. Preliminary experiments with the idle time of 20 workstations of the Seminar for Applied Mathematics SAM were carried out for testing purposes. The time necessary in this setting was estimated as 2 years – with luck; otherwise it could be 5 years as well. If the search were to be successful within a reasonable timespan, we could claim a world record that wouldn't be easy to break. Dense clusters of 19 primes in the patterns of $[13, \ldots, 89]$, $[37, \ldots, 113]$ or their mirror images are expected to repeat/occur only in the range of $10^{25}$ or even $10^{26}$. For the final "attack" on the 18er 432 processors of the Beowulf Cluster and 20 workstations of SAM were involved. The search has now been completed up to $2.9999949836 \cdot 10^{24}$. The first search was caried out with the pattern

$$\mathbf{c} = [x - 83,\ x - 79,\ x - 73,\ \ldots,\ x - 19,\ x - 17,\ x - 13] \tag{2}$$

consisting of 18 elements, such that for $x = 0$ the sequence of consecutive negative primes beginning at -83 and ending at -13 is obtained. For the corresponding Hardy-Littlewood constant the value $h_c = 6723654.312$ is obtained. Equ. (1) implies that the expected number of occurrences of the pattern $\mathbf{c}$ (as well as of its mirror image) in a large interval of length $\Delta$ near a much larger $x$ is approximately given by $h_c\,\Delta\,(\log x)^{-18}$. Integration of this average density yields the following expected frequencies $\mathrm{HL}(x)$ (referred to as the Hardy-Littlewood count, HL count for short):

| $x$ | $1_{10}24$ | $2_{10}24$ | $3_{10}24$ | $4_{10}24$ | $5_{10}24$ | $1_{10}25$ | $1_{10}26$ |
|---|---|---|---|---|---|---|---|
| $\mathrm{HL}(x)$ | 0.438 | 0.695 | 0.912 | 1.107 | 1.286 | 2.056 | 9.962 |

Therefore, one could "hope" that the above prime pattern $\mathbf{c}$ would repeat for some $x$ in the range of $3 \cdot 10^{24}$, and also that its mirror image occurs in the same range. One could even be lucky to have occurrences for smaller values of x, but also, with bad luck, the search might have to be pushed up

much higher. The table below summarizes some of the actions that lead to the discovery of the two 18-tuples of maximum density; for more details see [10], `Projects/cl18.pdf`.

| Pattern | Block | Begin/end of search | Date | Initial element | HLcount |
|---|---|---|---|---|---|
| [-83..-13] | 1 | 8/03 - 9/19/00 | | | 0.438 |
| [-83..-13] | 2 | 9/19 -10/26/00 | | | 0.695 |
| [-83..-13] | 3 | 10/29 -11/20/00 | 11/13/00 | 284537254250991186826807 | 0.880 |
| [ 13.. 83] | 1 | 12/19 - 1/23/01 | | | 0.438 |
| [ 13.. 83] | 2 | 1/23 - 2/27/01 | 1/31/01 | 190623083504664829329004 3 | 0.673 |
| [ 13.. 83] | 3 | 2/27 - 3/26/01 | | | 0.912 |

# 3 Distribution of a Particular 12-Tuple

In this section we describe the results of an extended study of the distribution of the particular $k$-tuple with $k = 12$, defined by the pattern

$$\mathbf{c} = [-23, -19, -17, -13, -11, -1, 1, 11, 13, 17, 19, 23]\,. \tag{3}$$

It is easily seen that $\mathbf{c}$ is admissible: modulo the primes $\leq 7$ the residue class 0 is vacant, modulo the primes $\geq 13$ the 12-tuple necessarily leaves at least one residue class empty, and modulo 11 the residue classes $\pm 4$ are empty. The pattern $\mathbf{c}$, spanning an interval of length 46, is not the densest admissible 12-tuple; the primes $[11, 13, \ldots, 53]$, spanning an interval of length 42 only, also form an admissible 12-tuple. The pattern $\mathbf{c}$, by its appealing symmetry and by the lack of an obvious reoccurrence in terms positive primes, promises to be an interesting object to study.

The prime $k$-tuple hypothesis predicts infinitely many prime instances of $\mathbf{c}$. By means of our algorithm ([10]: the Initial Report 2001, `/clprimes01.pdf`, and the experimental PARI code, `/paricode.gp`) all prime intances of $\mathbf{c}$ up to $1.000046276 \cdot 10^{21}$ were generated, 155 898 instances alltogether. Chinese remaindering was done with the first `np` $= 14$ primes. The primes up to 29 produce 52 224 tasks; the range mentioned above was covered with 5 sieve blocks of length `block` $= 15\,288$. These computations were performed during the testing phase of the algorithm on `Asgard` within about two weeks.

In the table below we report the initial primes of a few of the patterns generated: 16 at the beginning of the sequence, 7 at the current end, and 2 at each transition to a new decade.

| Index | Initial Prime | Index | Initial Prime |
|---|---|---|---|
| 1 | 41280160361347 | 205 | 99859425208272637 |
| 2 | 65073487398967 | 206 | 100306705532373247 |
| 3 | 273596722858597 | | |
| 4 | 305247832189207 | 996 | 998054166528529927 |
| 5 | 314546191059007 | 997 | 1004612024526939967 |
| 6 | 334701417639727 | | |
| 7 | 355340244780337 | 5330 | 9999309335149183957 |
| 8 | 552105775370287 | 5331 | 10001687719716943297 |
| 9 | 610954727181937 | | |
| 10 | 660119078678197 | 28104 | 99992682101800188517 |
| 11 | 767920116273217 | 28105 | 100004729076950887327 |
| 12 | 776137769447857 | | |
| 13 | 899316460923697 | 155892 | 999972884065149535357 |
| 14 | 957065931972967 | 155893 | 1000006962511995788257 |
| 15 | 1333086396411817 | 155894 | 1000017025199672760187 |
| 16 | 1490344945469287 | 155895 | 1000022902668956065807 |
| | | 155896 | 1000035195619662051757 |
| 43 | 9411514972889167 | 155897 | 1000037569658302164127 |
| 44 | 10012543630829647 | 155898 | 1000044549710737801177 |

In the following, we consider the counting function $\pi_{\mathbf{c}}(x)$, defined as the number of patterns $\mathbf{c}$ with initial prime $\leq x$. We will compare $\pi_{\mathbf{c}}(x)$ with the corresponding smooth counting function obtained by integrating the Hardy-Littlewood density given in Equ. (1). For the pattern $\mathbf{c}$ of Equ. (3) the Hardy-Littlewood constant

$$h_{\mathbf{c}} = 18867.657182534569614833826514092$$

is obtained. Instead of the conventional logarithmic integral

$$\mathrm{li}_k(x) := \int_2^x \frac{1}{(\log t)^k}\, dt$$

we prefer to use the generalization

$$R_k(x) := \sum_{j=0}^{\infty} \frac{(\log x)^j}{(k-1+j)!\; j\, \zeta(1+j)}\,, \quad k \in \mathbb{N}\,, \tag{4}$$

of Riemann's function $R(x) := R_1(x)$. Here $\zeta(s)$ is the Riemann zeta function, and for $j = 0$ the product $j\,\zeta(1+j)$ must be understood in the sense of the limit $\lim_{j\to 0}(j\,\zeta(1+j)) = 1$.

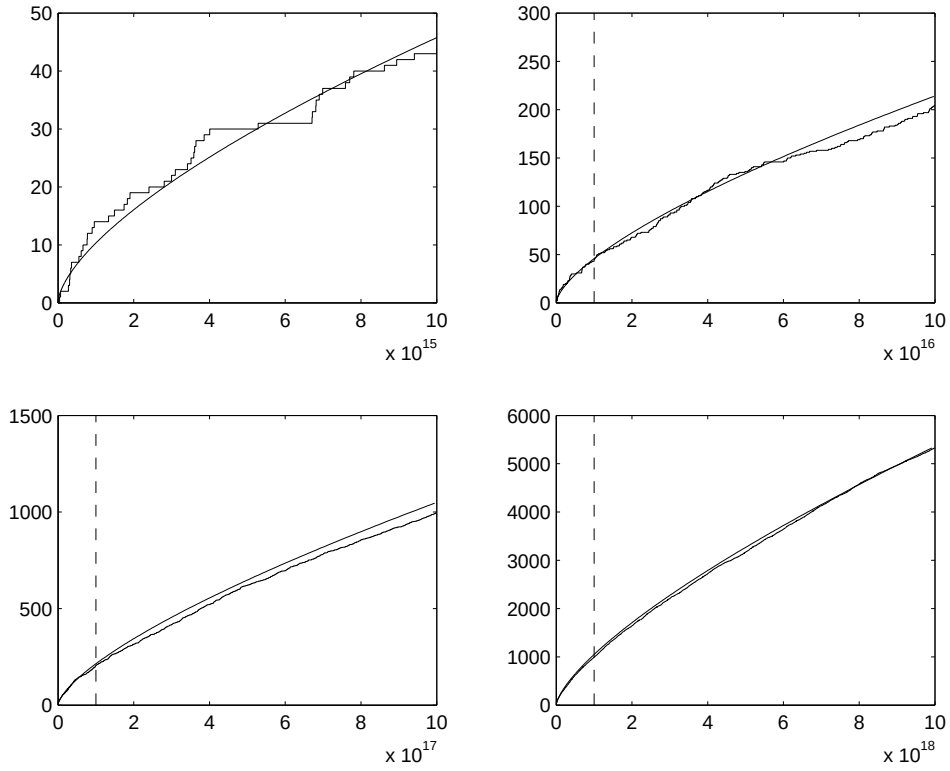


Figure 1: Counting function $\pi_{\mathbf{c}}$ and the smooth graph of $\mathrm{HL}(x)$ versus $x$. Range $0 < x < 10^j,\ j = 16, \ldots, 19$.

For $x \to \infty$ $R_k(x)$ and $\mathrm{li}_k(x)$ have asymptotically the same values. The advantage of $R_k(x)$ is the simple evaluation algorithm based on the series (4) and the regularity at $x = 1$. $R(x) = R_1(x)$ is known to be a very elegant monotonic approximation of the prime counting function $\pi(x)$. In all our examples the (modified Hardy-Littlewood) counting function

$$\mathrm{HL}(x) := h_{\mathbf{c}} \cdot R_k(x) \quad \text{with} \quad k = |\mathbf{c}| \tag{5}$$

agrees surprisingly well with $\pi_{\mathbf{c}}$, for large values of $x$ as well as for small ones.

In the six frames of Fig. 1, Fig. 2, Fig. 3 we plot $\pi_{\mathbf{c}}$ and $\mathrm{HL}(x)$ in the regions $0 < x < 10^j,\ j = 16, \ldots, 21$. The range of $x$ in the subsequent frame is increased 10-fold. The preceding frame appears to the left of the dashed lines in compressed form.

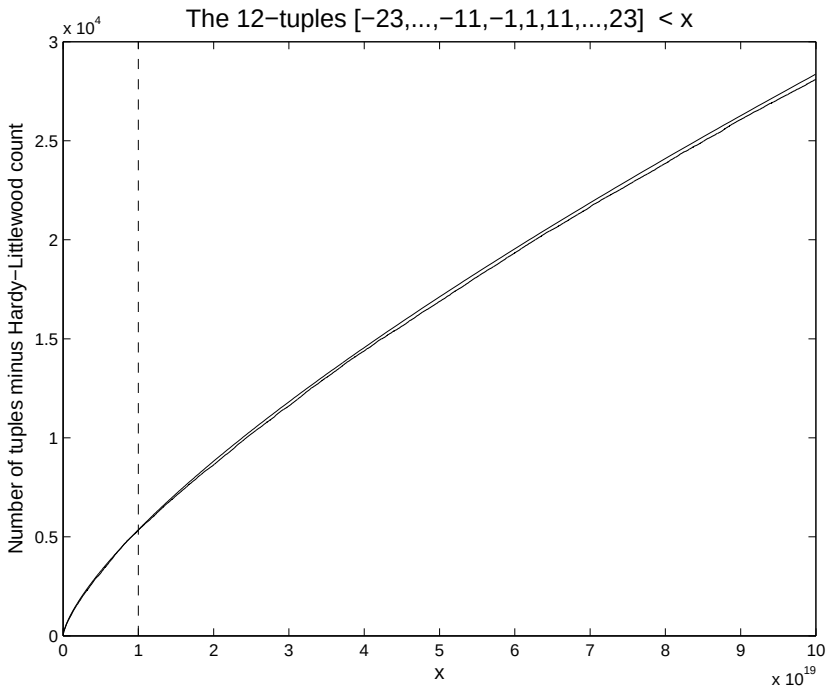


Figure 2: Counting function $\pi_{\mathbf{c}}$ and the smooth graph of HL($x$) versus $x$. Range $0 < x < 10^{20}$. $\pi_c$ below HL.

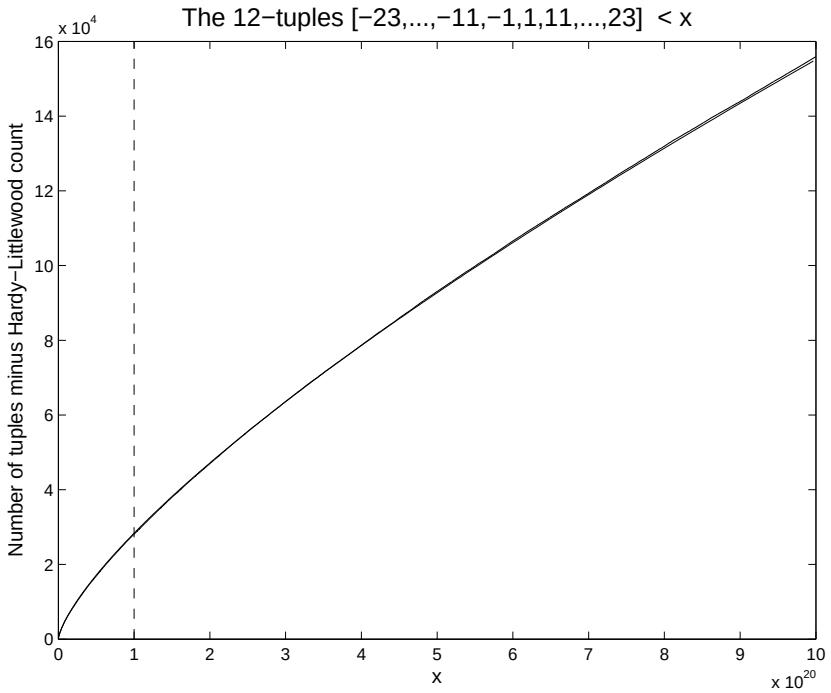


Figure 3: Counting function $\pi_{\mathbf{c}}$ and the smooth graph of HL($x$) versus $x$. Range $0 < x < 10^{21}$. Crossover of $\pi_{\mathbf{c}}$.

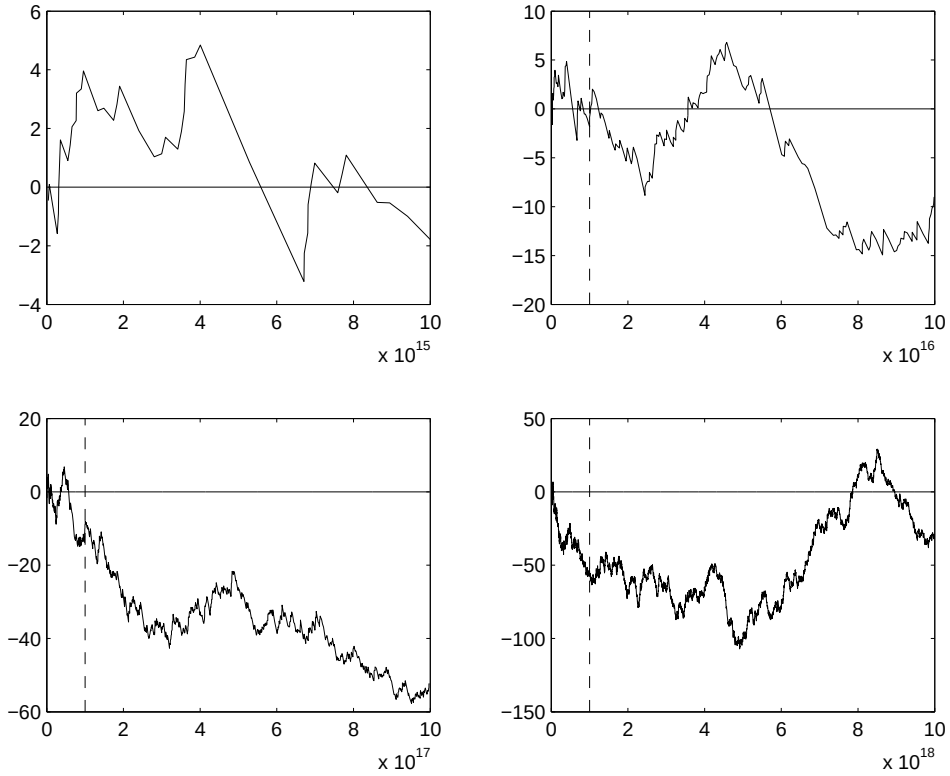

Figure 4: Difference $\pi_{\mathbf{c}} - \mathrm{HL}(x)$ versus $x$. Range $0 < x < 10^j,\ j = 16, \ldots, 19$.

In the six frames of Fig. 4, Fig. 4, Fig. 6 we plot the difference $\pi_{\mathbf{c}} - \mathrm{HL}(x)$ versus $x$ in the same regions $0 < x < 10^j,\ j = 16, \ldots, 21$. Again, the range of $x$ in the subsequent frame is increased 10-fold. The preceding frame appears to the left of the dashed lines in compressed form.

Despite apparent discrepancies between $\pi_{\mathbf{c}}$ and $\mathrm{HL}(x)$ (e.g. in the range $10^{20}$, Fig. 5) the agreement of the two graphs over the full range of 7 powers of 10 is striking (Fig. 3). Whereas the prime counting function $\pi(x)$ oscillates on a short scale, the counting functions of prime twins and longer clusters of primes seem to depart from the average in large-scale excursions. E.g., the pattern $\mathbf{c}$ of Equ. (3) is deficient in $9 \cdot 10^{18} < x < 2.7 \cdot 10^{20}$, i.e. the bounds of this interval differ by as much as a factor of 30. Even in the simplest case, the prime twins, the source of such large-scale anomalies is still unknown and is the subject of theoretical research.

In Fig. 7, the final graphic of this section, we summarize the properties of $\pi_{\mathbf{c}}$ by using a logarithmic scale in $x$ and plotting the normalized difference

$$D(x) = \frac{\pi_{\mathbf{c}} - \mathrm{HL}(x)}{\sqrt{\mathrm{HL}(x)}}\,.$$

In the range considered this choice of the normalizing factor seems to keep the amplitude roughly constant; however, its true growth rate is not known.

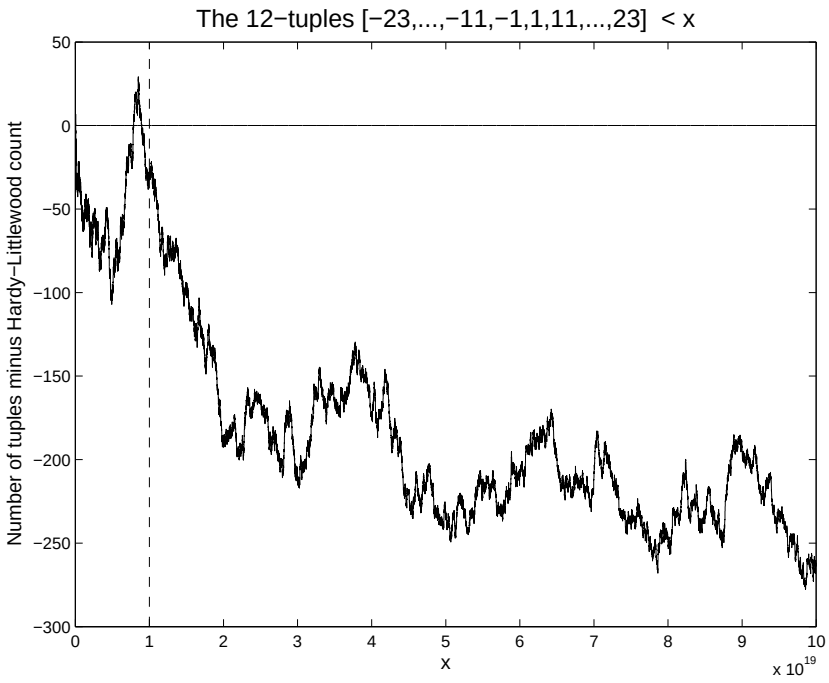


Figure 5: Difference $\pi_{\mathbf{c}} - \mathrm{HL}(x)$ versus $x$. Range $0 < x < 10^{20}$.

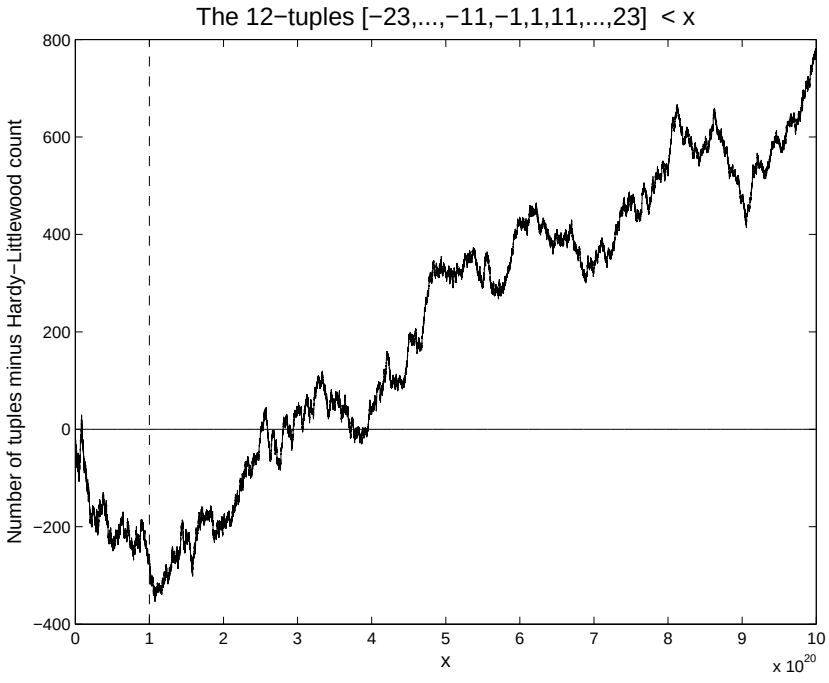


Figure 6: Difference $\pi_{\mathbf{c}} - \mathrm{HL}(x)$ versus $x$. Range $0 < x < 10^{21}$.

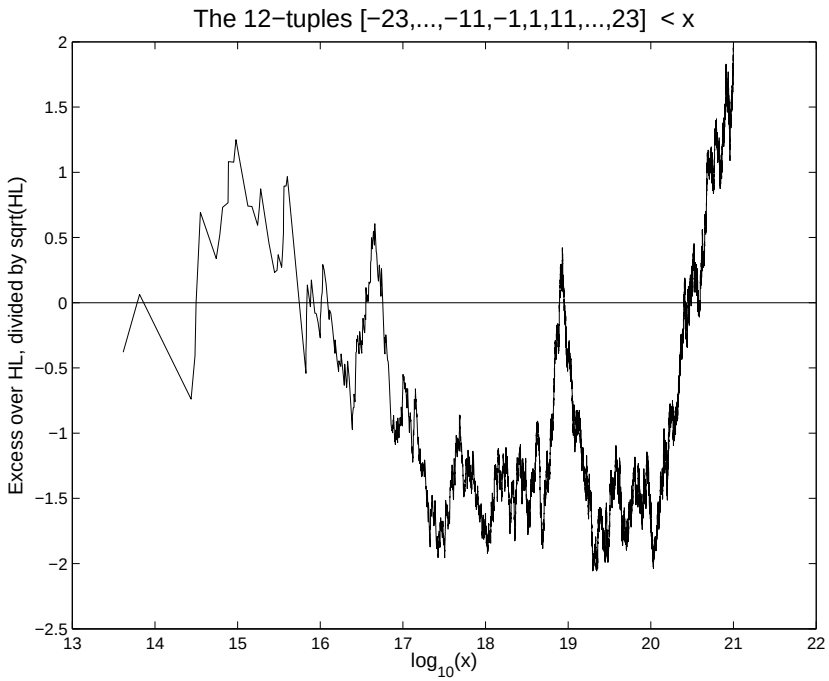


Figure 7: Normalized difference $(\pi_{\mathbf{c}} - \mathrm{HL}(x))/\sqrt{\mathrm{HL}(x)}$ versus $\log_{10}(x)$. Range $10^{14} < x < 10^{21}$, logarithmic scale.

# 4 Some Particular Patterns

## 4.1 Densest 16-Tuples

The densest 16-tuples occur in two patterns which may be defined by sequences of consecutive small primes, namely

$$\mathbf{c} = [13, 17, \ldots, 71, 73] \quad \text{and} \quad \mathbf{c}' = [-73, -71, \ldots, -17, -13]\,,$$

their common Hardy-Littlewood constant being $h_{\mathbf{c}} = 751221.42544528571$. Below we will compare the modified Hardy-Littlewood counting function

$$\mathrm{HL}(x) = h_{\mathbf{c}} \cdot R_{16}(x)$$

with the actual counting functions $\pi_{\mathbf{c}}(x)$, $\pi_{\mathbf{c}'}(x)$ deduced from the 67 instances present in $0 < x < 10^{23}$.

| | Pattern [13,...,73] | Pattern [-73,...,-13] |
|---|---|---|
| 1 | 13 | 47710850533373130107 |
| 2 | 695874886175252911063 | 347709450746519734877 |
| 3 | 1567582627835236839763 | 1099638576123052218257 |
| 4 | 1750052554011927712483 | 1169914227530138703617 |
| 5 | 2257588388550898970503 | 1522014304823128379267 |
| 6 | 3789227751026345304613 | 1620784518619319025977 |
| 7 | 4654682384109074514133 | 2639154464612254121537 |
| 8 | 5022156579757255625623 | 3259125690557440336637 |
| 9 | 1359923609915916655303 3 | 9042634271485192050677 |
| 10 | 2989452282236368465210 3 | 9239395687646993061197 |
| 11 | 3571890454453671544888 3 | 1557105375804829330780 7 |
| 12 | 4242118368555274746232 3 | 2062814905069869466816 7 |
| 13 | 4762441549049876396398 3 | 2094735361787781029617 7 |
| 14 | 5006982385004903663053 3 | 2318216050595492578831 7 |
| 15 | 5629492678618056950395 3 | 2781411605490120058756 7 |
| 16 | 6087785151809085811780 3 | 3040614966934934146057 7 |
| 17 | 6687113537995314861130 3 | 3160738342468239408175 7 |
| 18 | 7074349136652952646185 3 | 3425473051196115882262 7 |
| 19 | 7203955544120235485278 3 | 4067597341184059781398 7 |
| 20 | 7293916977856497889594 3 | 4145915926465574091130 7 |
| 21 | 7831416773806452904771 3 | 4979800221504777322954 7 |
| 22 | 7941582164381850539248 3 | 5328465814044162225736 7 |
| 23 | 8075592481945860598420 3 | 6119681340693355417282 7 |
| 24 | 8143399554377482077376 3 | 6256456596553113881230 7 |
| 25 | 8340568798040699893366 3 | 6708963543060110455423 7 |
| 26 | 9309135549951197949682 3 | 7337749332281935708441 7 |
| 27 | 9404598641903715852295 3 | 7535649850875717733462 7 |
| 28 | 9497586673559596608783 6 | 8044400115412305232888 7 |
| 29 | 9790764647833655281489 3 | 8104457676868698466621 7 |
| 30 | 9951746792515376452297 3 | 8206956846198290956075 7 |
| 31 | | 8604271520263483266502 7 |
| 32 | | 8807704495712056982347 7 |
| 33 | | 8917249345185727510264 7 |
| 34 | | 9157616573448498222367 7 |
| 35 | | 9395429535355424159899 7 |
| 36 | | 9769236954161084480380 7 |
| 37 | | 9865959372187038930109 7 |

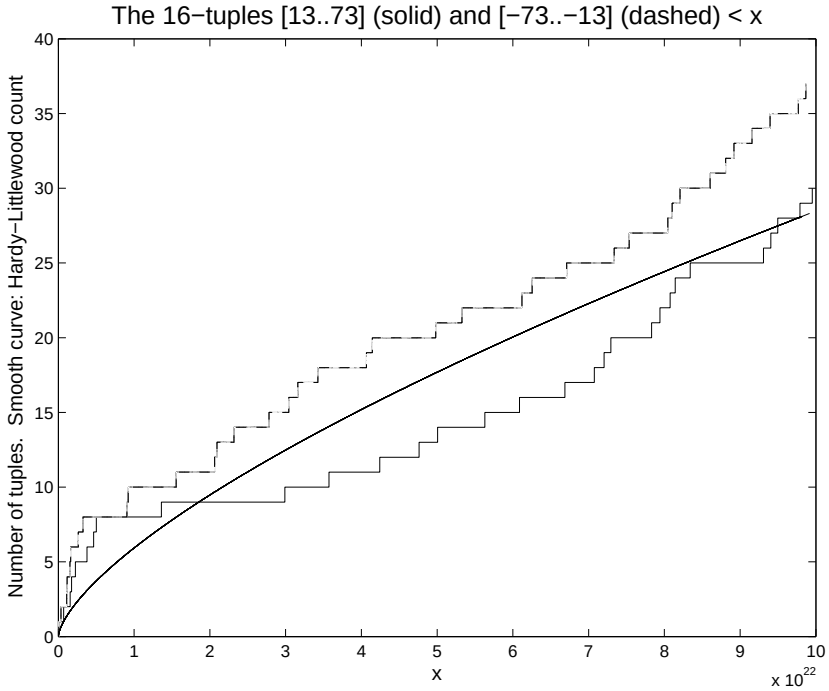


Figure 8: The densest 16-tuples in the range $0 < x < 10^{23}$ and their HL count. Solid: Pattern [13,...,73], dashed: Pattern [-73,...,-13].

## 4.2 Two Octuples at Minimum Distance

The densest octuples occur in three patterns which may all be defined by sequences of consecutive small primes, namely

$$\mathbf{c_1} = [17, \ldots, 43]\,,\ \mathbf{c_2} = [11, \ldots, 37]\,,\ \mathbf{c_2}' = [-37, \ldots, -11]\,.$$

$\mathbf{c_1}$, spanning a range of 26, is the longest symmetric pattern of maximum density. We observe that the 16-tuple $\mathbf{C}$ consisting of two copies of $\mathbf{c_1}$, concatenated at the distance of 34,

$$\mathbf{C} = [-43, -41, -37, -31, -29, -23, -19, -17, 17, 19, 23, 29, 31, 37, 41, 43]\,,$$

is admissible (see the remark at the beginning of Section 3). For the corresponding Hardy-Littlewood constant $h_{\mathbf{C}}$ we obtain

$$h_{\mathbf{C}} = \frac{13312}{2625} \cdot h_{\mathbf{c}} = 3809622.71067719746\,,$$

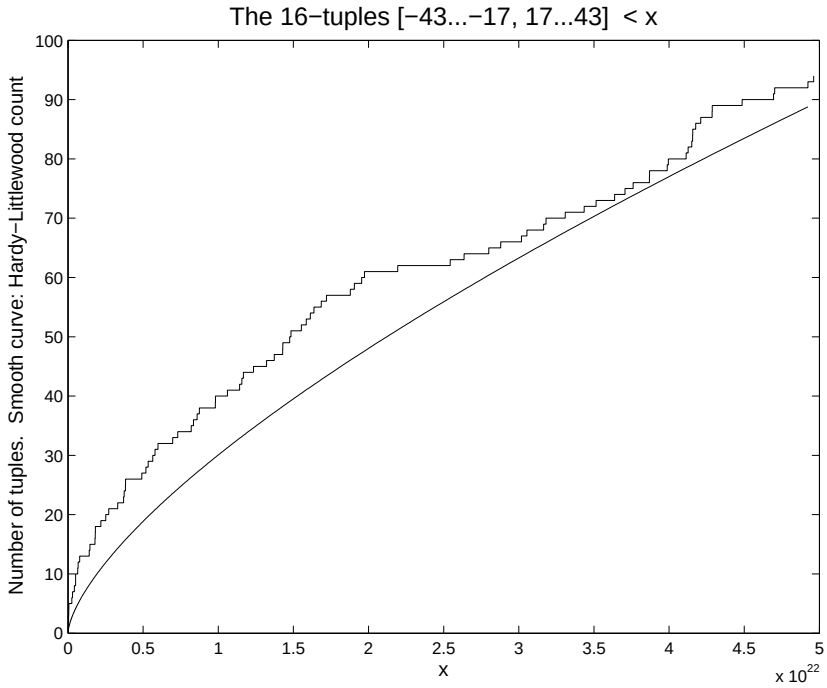


Figure 9: 94 pairs of octuples in the range $0 < x < 5 \cdot 10^{22}$ and their HL count, Pattern [-43,...,-13, 13,...,43].

where $h_{\mathbf{c}}$ is defined in Section 4.1. Up to the limit $X = 5 \cdot 10^{22}$ we expect $\mathrm{HL}(X) = h_{\mathbf{C}} \cdot R_{16}(X) = 89.7$ occurrences of the pattern $\mathbf{C}$. The distribution of the 94 prime instances of $\mathbf{C}$ is shown in Fig. 9.

The patterns

$$\mathbf{C_1} = [-43, -41, -37, -31, -29, -23, -19, -17, -1, 1, 17, 19, 23, 29, 31, 37, 41, 43, 47]$$
$$\mathbf{C_2} = [-47, -43, -41, -37, -31, -29, -23, -19, -17, -13, 1, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47]$$

of 18 or 21 elements, respectively, both containing $\mathbf{C}$ as a subset, are found to be admissible as well. Therefore, it is expected that some of the 94 instances of $\mathbf{C}$ will actually be subsets of $\mathbf{C_1}$ or $\mathbf{C_2}$ with more than 16 elements. However, instances of $\mathbf{C_1}$ or $\mathbf{C_2}$ themselves are expected to occur only in the range of $10^{24}$ or $10^{30}$, respectively. In the table below we list the initial primes of all 94 instances of $\mathbf{C}$, together with additional elements from $\mathbf{C_1}$ or $\mathbf{C_2}$ contained in the patterns, if present (in the columns `more`). No patterns with more than 18 elements exist in the range considered.

| Index | Initial Prime | more | Index | Initial Prime | more |
|---|---|---|---|---|---|
| 1 | 10458834002271815117 | | 48 | 14284612658991181216667 | |
| 2 | 26476006821087640697 | -47 | 49 | 14297146316095286150387 | |
| 3 | 44350865905809142637 | | 50 | 14754208002759013618787 | |
| 4 | 54014646858393564377 | -47,13 | 51 | 14841953373654341817917 | -13 |
| 5 | 62155369550078511587 | -47 | 52 | 15532484963882491131407 | -13 |
| 6 | 253586253591518370557 | -47 | 53 | 15853064083490265705107 | 47 |
| 7 | 304079924911990894547 | 13 | 54 | 16130615338665630649727 | |
| 8 | 423291158347150012877 | | 55 | 16372205147465011401437 | |
| 9 | 511505988322414165037 | | 56 | 16858454028003145410887 | |
| 10 | 512761727903842750367 | | 57 | 17198991227897242847117 | |
| 11 | 644424770171034352457 | | 58 | 18786403726068543935327 | |
| 12 | 675759858713748355427 | | 59 | 19054062004470982522067 | 13 |
| 13 | 780362378270548056017 | | 60 | 19547070837095799260297 | |
| 14 | 1416058157129915879537 | | 61 | 19728243594481889687177 | |
| 15 | 1457922032650513232837 | -47 | 62 | 21944528505231528596357 | |
| 16 | 1792491363005354644667 | | 63 | 25435635163943029705637 | 1 |
| 17 | 1812456008867090693987 | | 64 | 26360513714179113726137 | |
| 18 | 1826785107639242841047 | | 65 | 28001420822911641278207 | |
| 19 | 2193719983731075106187 | | 66 | 28794590387631746120477 | |
| 20 | 251993592777795291137 | | 67 | 30178895046554873882297 | |
| 21 | 271548515291274688627 | | 68 | 30544994855724415219187 | |
| 22 | 330993572459771675449 7 | | 69 | 31647354432491850512417 | -1 |
| 23 | 370304812855698769351 7 | | 70 | 31810951085696093237807 | |
| 24 | 374163604739166991744 7 | -47,-1 | 71 | 33086235745772630878337 | |
| 25 | 382477812047629787176 7 | | 72 | 34351709846301758103227 | |
| 26 | 382940205077341104496 7 | | 73 | 35138861572319523491507 | |
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| 37 | 859170389467474625536 7 | 47 | 84 | 41564901882588422431127 | -47 |
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| 39 | 980689407971194660426 7 | 1 | 86 | 41773491218296251541007 | |
| 40 | 981534301482574919767 7 | | 87 | 42108300110425746630467 | |
| 41 | 1060632573416848306870 7 | -47 | 88 | 42858583357347352501787 | |
| 42 | 1142334742363430606986 7 | | 89 | 42876198440221191818807 | |
| 43 | 1156868083571065604041 7 | | 90 | 44860553600587843401317 | |
| 44 | 1167355630652442525289 7 | 1 | 91 | 46952529647970814520687 | |
| 45 | 1234487494052514866467 7 | | 92 | 47030198166588987767297 | |
| 46 | 1321641183610606611055 7 | | 93 | 49235705631942915972767 | |
| 47 | 1372929723036365437939 7 | 47 | 94 | 49623872890924546023767 | |

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