Advances in Stochastic Portfolio Theory
Catalogue of Exam Questions

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QUESTIONS[1]:

1. Outline the modeling framework for the stock price process as set forth in Definition 1.1.1. and ensuing calculations and explain the conditions imposed on the growth rate and the volatility processes.

2. Characterize a stock market and its instantaneous covariance structure. What important properties of the market are introduced on basis of the covariance matrix and how can they be interpreted (cf. Def. 1.1.2.)?

3. Define a portfolio of stocks in the market model and retrieve the characterizations of the instantaneous return of the portfolio and the log-portfolio value process (cf. Def. 1.1.4. and Prop. 1.1.5.).

4. How may dividend rates be incorporated in the given market model (cf. Def. 1.1.9. and ensuing calculations)?

5. Specify relative returns of a stock or a portfolio relative to a benchmark portfolio and discuss the relative covariance matrix and its properties. Further, describe the market portfolio as canonical benchmark (cf. Def. 1.2.2., 1.2.3. and 1.2.4., Lemma 1.2.2.).

6. Characterize the numéraire invariance of the excess growth rate (cf. Lemma 1.3.4.).

7. Explain the term *market coherence* and discuss its effect in terms of individual stocks' and the market's growth rate (cf. Def. 2.1.1. and Prop. 2.1.2.).

[1] N.b.: all references are given with respect to Fernholz, E.R.: *Stochastic Portfolio Theory*. Springer Verlag, Berlin/Heidelberg/New York, 2002.

8. Sketch, which theoretical results on outperforming the market may be obtained in a coherent setting (cf. Prop. 2.1.9.).

9. Describe the concept of *stock market diversity* and its economic motivation. What can be said about the market's excess growth rate and individual stocks' growth rates if diversity prevails (cf. Def. 2.2.1., Prop. 2.2.2. and 2.2.3.)?

10. Describe under which circumstances market diversity will prevail in a market where all stocks have the same augmented growth rate, i.e. including dividends (cf. Prop. 2.2.8. and Cor. 2.2.9.).

11. Outline, how the entropy function may be used to measure market diversity and functionally generate a portfolio (cf. Sect. 2.3).

12. Characterize the concept of portfolio generating functions and the properties of the resulting portfolio relative to the market (cf. Def. 3.1.1., Prop. 3.1.3. and Thm. 3.1.5.).

13. Calculate the portfolio weights and the drift process for the portfolio generated by the function $S(x) = 1 - \frac{1}{2}\sum_{i=1}^{n} x_i^2$. What can be said about its behavior relative to the market and how may one exploit such a relative arbitrage opportunity (cf. Ex. 3.3.3.)?

14. Explain the concept of relative arbitrage and dominating portfolios. What can be said about the necessary time horizon to exploit arbitrage opportunities (cf. Sect. 3.3.)?

15. Characterize semimartingale local times and outline the results presented in the lectures (cf. Def. 4.1.1. and following notions, Lemma 4.1.9.).

16. Describe the dynamics of ranked market weights and explain the significance of local times in the characterization (cf. Def. 4.0.1., Def. 4.1.1. and Prop. 4.1.11.).

17. How may the concept of functionally generated portfolios be extended to ranked market weights (cf. Thm. 4.2.1.)?

18. Sketch the structure of the capital distribution of the market and characterize the notion of asymptotic stability of a market (cf. Sect. 5.1. and Def. 5.3.1.).

19. Explain, how a stable approximation for the dynamics of ranked market weights may be obtained in an asymptotically stable market (cf. Prop. 5.3.2. and ensuing calculations).

20. Outline the Atlas model and describe its characteristics (cf. Ex. 5.3.3.).