Asymptotic Proportion of Arbitrage Points in Fractional Binary Markets

Abstract

A fractional binary market is an approximating sequence of binary models for the fractional Black–Scholes model, which Sottinen constructed by giving an analogue of the Donsker's theorem. In a binary market the arbitrage condition can be expressed as a condition on the nodes of the binary tree. We call "arbitrage points" the points in the binary tree which verify such an arbitrage condition and "arbitrage paths" the paths in the binary tree which cross at least one arbitrage point. Using this terminology, a binary market admits arbitrage if and only if there is at least one arbitrage point in the binary tree or equivalently if there is at least one arbitrage path. Following the lines of Sottinen, who showed that the arbitrage persists in the fractional binary market, we further prove that starting from any point in the tree, we can reach an arbitrage point. This gives information about the structure of the set of arbitrage points and implies that its cardinal is asymptotically infinite. The main results of our work are related with the asymptotic proportion of arbitrage points and of arbitrage paths in the fractional binary market. We first observe that the parameters of the fractional binary models verify a scaling property. This permits to characterise the proportion of arbitrage points in terms of a rescaled random walk. By studying the convergence properties of this rescaled random walk, we obtain the asymptotic proportion of arbitrage points. We moreover show that, when H is close to 1, with probability 1 a path in the binary tree crosses an infinite number of arbitrage points. In particular, for such H, the asymptotic proportion of arbitrage paths is equal to 1. The talk is based on ongoing work with Irene Klein and Lavinia Perez-Ostafe.