

Using a quadratic IP:

$$f'(x_0) \approx p'[f|x_{-1}, x_0, x_1] = \frac{f(x_1) - f(x_{-1})}{x_1 - x_{-1}}$$

$$= \frac{f(x_0+h) - f(x_0-h)}{2h}$$

$$f''(x_0) \approx p''[f|x_{-1}, x_0, x_1] = \frac{f(x_1) - 2f(x_0) + f(x_{-1}))}{h^2}$$

$$= \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

(so-called centered FD)

Ex.: (5) Approx. first derivative of $f(x) = \sin(x)$
at $x = 1.2$ (exact $f'(x) = \cos(x)$ of course :-)

→ slides

We observe

(i) The error $e = |p'[f|\dots] - f'(x)|$ behaves
as $\begin{cases} O(h) \\ O(h^2) \end{cases}$ for $\begin{cases} \text{forward/backward} \\ \text{centered} \end{cases}$ FD

(ii) The error grows if h is too small on computer
Why? Due to the finite precision of (floating point) numbers