The error estimates could be derived from the interpolation error... But there is another way with the help of Taylor expansions/series:

$$f(x+h) = f(x) + f'(x)\cdot h + \frac{f''(x)}{2} h^2 + \dots + \frac{f^{(k)}(x)}{k!} h^k + \frac{f^{(k+1)}(\xi)}{(k+1)!} h^{k+1}$$

for some $\xi \in [x, x+h]$

remainder term

(sometimes error term)

<u>Ex.:</u> (6) forward FD approx of f':

$$\frac{f(x_0+h) - f(x_0)}{h} = \frac{\cancel{f(x_0)} + f'(x_0)\cdot h + \frac{f''(x_0)}{2} h^2 + \dots - \cancel{f(x_0)}}{h}$$

$$= f'(x_0) + \frac{h}{2} f''(x_0) + \dots \quad ?$$

$$= f'(x_0) + \mathcal{O}(h) \quad ✓.$$

Negligible for h small: $h > h^2 > h^3 > \dots$

So forward FD has order of accuracy $r = 1$. They are first order accurate