Def.: A fixed-point equation is said to be consistent with the original equation if

$$f(x^*) = 0 \iff x^* = \phi(x^*)$$

Ex.: (1) Solve $f(x) = x \cdot e^x - 1 = 0$ for $x \in [0,1]$

(i) $x \cdot e^x - 1 = 0$

$$x \cdot e^x = 1$$

$$x = e^{-x} = \phi_1(x) \quad \text{fixed-point eq.}$$

(consistent ✓.)

(ii) $x = \phi_2(x)$ with $\phi_2(x) = \dfrac{x^2 \cdot e^x + 1}{e^x(1+x)}$

(iii) $x = \phi_3(x)$ with $\phi_3(x) = x + 1 - x \cdot e^x$

$\rightsquigarrow$ Slides

... Exercise: show consistency!

Rem.: (i) Fixed-point iterations not unique

(ii) Fixed-point iterations may not converge

(iii) If they converge, they may do that with different speeds