

Def.: A sequence $x^{(k)}$ with limit x^* converges with order $p \geq 1$, if there exists a constant $C > 0$ such that

$$|x^{(k+1)} - x^*| \leq C |x^{(k)} - x^*|^p$$

for all sufficiently large k .

For $p=1$, it must $0 < C < 1$.

The constant C is called the rate of convergence.

In particular, convergence with order $\begin{cases} p=1 \\ p=2 \end{cases}$

is called $\begin{cases} \text{linear} \\ \text{quadratic} \end{cases}$.

It is often helpful, e.g. for code verification, to measure C and p in numerical experiments.

For this we define the error at the k -th iteration as

$$E^{(k)} = |x^{(k)} - x^*|$$