II.2.1 Newton's method

Idea: linearize $\vec{f}$

Jacobian Matrix

$$D f(\vec{x}) = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{pmatrix}$$

Taylor at $\vec{x}^{(k)}$

$$\underbrace{\vec{f}(\vec{x}) = \vec{f}(\vec{x}^{(k)}) + D\vec{f}(\vec{x}^{(k)})(\vec{x} - \vec{x}^{(k)})}_{\tilde{\vec{f}} = \vec{f}(\vec{x}^{(k)}) + D\vec{f}(\vec{x}^{(k)})(\vec{x} - \vec{x}^{(k)}) \overset{!}{=} 0} + \dots$$

$$\rightsquigarrow \quad \vec{x}^{(k+1)} = \vec{x}^{(k)} - D\vec{f}(\vec{x}^{(k)})^{-1} \, \vec{f}(\vec{x}^{(k)})$$

inverse of Jacobian