

By defining

$$\vec{z}(t) = \begin{pmatrix} z_0(t) \\ z_1(t) \\ \vdots \\ z_{n-2}(t) \\ z_{n-1}(t) \end{pmatrix} \quad \text{and} \quad \vec{g}(t, \vec{z}) = \begin{pmatrix} z_1(t) \\ z_2(t) \\ \vdots \\ z_{n-1}(t) \\ f(t, z_0(t), \dots, z_{n-1}(t)) \end{pmatrix}$$

we get a first order system of ODEs

$$\dot{\vec{z}}(t) = \vec{g}(t, \vec{z}(t))$$

Together with the IVs we get a first order IVP.

Ex.: (S) $\ddot{y} = \dot{y} + y^2 - e^t$, $t \in [t_0, T]$

$$y(t_0) = 1, \quad \dot{y}(t_0) = 0$$

Reduction to first order:

$$z_0(t) = y(t)$$

$$z_1(t) = \dot{y}(t) = \dot{z}_0(t)$$

$$z_2(t) = \ddot{y}(t) = \dot{z}_1(t)$$

$$= \dot{y}(t) + y^2(t) - e^t$$

$$= z_1(t) + z_0^2(t) - e^t$$