

III.3 Error estimation and convergence

To analyze the accuracy of so-called one-step methods we consider the general expression

$$y_{j+1} = y_j + h \cdot \phi(t_j, y_j, y_{j+1}, h)$$

and call ϕ the increment function.

Rem.: (i) $\phi(t_j, y_j, y_{j+1}, h) = f(t_j, y_j) \quad \text{EE}$

(ii) $\phi(t_j, y_j, y_{j+1}, h) = f(t_{j+1}, y_{j+1}) \quad \text{IE}$

Def.: the local truncation error (LTE) at t_j is defined by

$$e_j = \underset{\substack{\uparrow \\ \text{exact sol. at } t_j}}{y(t_j)} - \left(\underset{\substack{\uparrow \\ t_{j-1}}}{y(t_{j-1})} + h \cdot \phi(t_{j-1}, y(t_{j-1}), y(t_j), h) \right)$$

The LTE is the error after one step of a method executed with the exact solution.

Def.: the global truncation error (GTE) at t_j is defined by

$$E_j = \underset{\substack{| \\ \text{exact}}}{y(t_j)} - \underset{\substack{| \\ \text{approx. solution at } t_j}}{y_j}$$