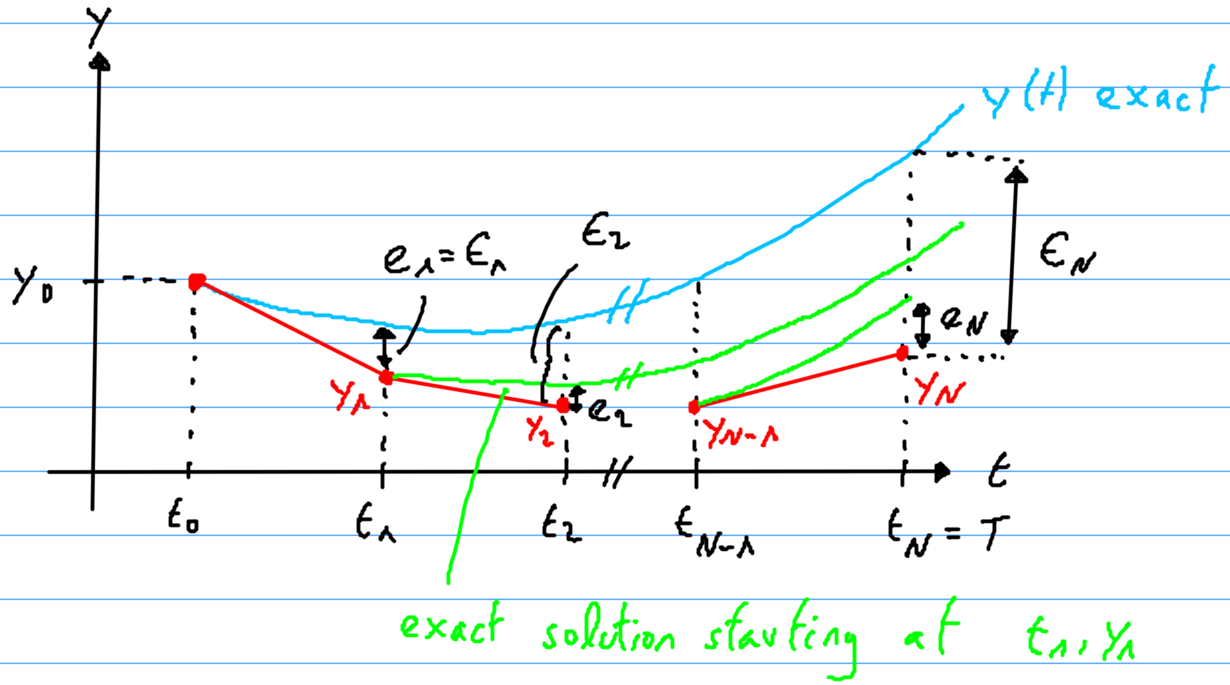
Graphically:



The LTE can "easily" be obtained by a Taylor expansion:

<u>Ex.:</u> (7) LTE of EE $\phi(t_j, y_j, y_{j+1}, h) = f(t_j, y_j)$

$$e_j \overset{\text{Def.}}{=} y(t_j) - \Big(y(t_{j-1}) + h \cdot \phi\big(t_{j-1}, y(t_{j-1}), y(t_j)\big) \Big)$$

$$= y(t_j) - y(t_{j-1}) - h \cdot f\big(t_{j-1}, y(t_{j-1})\big)$$

$$= \overbrace{y(t_{j-1}+h)}^{\text{Taylor}} - y(t_{j-1}) - h \cdot f\big(t_{j-1}, y(t_{j-1})\big)$$

$$= \cancel{y(t_{j-1})} + \cancel{h \cdot \underbrace{\dot{y}(t_{j-1})}_{\text{ODE } f(t_{j-1}, y(t_{j-1}))}} + \frac{h^2}{2}\ddot{y}(t_{j-1}) + \frac{h^3}{6}\dddot{y}(t_{j-1}) + \ldots$$

$$- \cancel{y(t_{j-1})} - \cancel{h \cdot f\big(t_{j-1}, y(t_{j-1})\big)}$$

$$= \frac{h^2}{2}\ddot{y}(t_{j-1}) + h^3 \cdot (\cdots) + h^4 \cdot (\cdots) + \ldots = \mathcal{O}(h^2)$$