<u>Ex.:</u> (8) LTE of IE $\phi(t_j, y_j, y_{j+1}, h) = f(t_{j+1}, y_{j+1})$

$$e_j \overset{\text{Def.}}{=} y(t_j) - y(t_{j-1}) - h \cdot f(t_j, y(t_j))$$

$$= \dots \qquad \color{red}{y(t_j - h) = \dots \text{ Taylor} \dots}$$

$$= \mathcal{O}(h^2)$$

<u>Rem.:</u> The LTE depends on the smoothness of the solution (e.g. $\ddot{y}$ above!) and therefore on the smoothness of the right-hand side function $f(t, y(t))$:

$$\ddot{y}(t) = \frac{d}{dt}\left(\dot{y}(t)\right)$$

$$\overset{\text{ODE}}{=} \frac{d}{dt}\left(f(t, y(t))\right)$$

$$= \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} \underbrace{\frac{dy}{dt}}_{\text{ODE } \dot{y} = f(t,y)} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial y} f$$

So we have seen that the LTE error is easy to compute with Taylor series ...

How is the LTE related to the practically more meaningfull GTE ?