III.5 Absolute stability

Let's consider the very simple IVP

$$\dot{y}(t) = \lambda\, y(t)$$
$$y(0) = y_0 \qquad (\lambda \in \mathbb{R} \text{ or } \mathbb{C})$$

which is known as the Dahlquist test equation (DTE) (in the present context).

The analytical solution is simply

$$y(t) = y_0\, e^{\lambda t}$$

Let's apply EE to this IVP

$$
\begin{aligned}
y_{j+1} &= y_j + h \cdot f(t_j, y_j) \\
&= y_j + h\lambda y_j \\
&= (1 + h\lambda)\, y_j \qquad y_j = (1+h\lambda)\, y_{j-1} \\
&= (1 + h\lambda)^2\, y_{j-1} \qquad y_{j-1} = (1+h\lambda)\, y_{j-2} \\
&\;\vdots \\
&= (1 + h\lambda)^{j+1}\, y_0
\end{aligned}
$$