

In ex. (16) $S = \frac{1000}{1/2} = 2000$

Stiffness is also very common in nonlinear systems of ODEs

$$\dot{\vec{y}}(t) = \vec{F}(t, \vec{y}(t))$$

A local measure of stiffness is obtained by linearizing the system of ODEs at some point (of interest!) t_n, \vec{y}_n :

$$\begin{aligned} \vec{F}(t, \vec{y}) \approx & \vec{F}(t_n, \vec{y}_n) + \frac{\partial \vec{F}}{\partial t}(t_n, \vec{y}_n) \cdot (t - t_n) \\ & + \underbrace{\frac{\partial \vec{F}}{\partial \vec{y}}(t_n, \vec{y}_n)}_{\text{Jacobian matrix } J = \frac{\partial \vec{F}}{\partial \vec{y}}} \cdot (\vec{y} - \vec{y}_n) \end{aligned}$$

So one obtains the following inhomogeneous linear system of ODEs

$$\dot{\vec{y}}(t) = \underbrace{J(t_n, \vec{y}_n)}_A \vec{y}(t) + \underbrace{\left(\vec{F}(t_n, \vec{y}_n) + \frac{\partial \vec{F}}{\partial t}(t_n, \vec{y}_n) \cdot (t - t_n) - J(t_n, \vec{y}_n) \vec{y}_n \right)}_{\vec{b}(t)}$$