IV. Partial Differential Equations

Practical problems often depend on more than one variable, e.g. time (t) and space (x, y, z).
This leads to so-called Partial Differential Equations (PDEs).

Classical examples:

(i) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$ Laplace eq.

(ii) $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = f(x,y)$ Poisson eq.

(iii) $\dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2}$ Heat eq. (Diffusion)

(iv) $\dfrac{\partial^2 u}{\partial t^2} - \dfrac{\partial^2 u}{\partial x^2} = 0$ Wave eq.

In order to form well-posed problems, PDEs have to be supplemented by appropriate boundary conditions and sometimes also with initial conditions.

The above (linear) PDEs can be classified as elliptic, parabolic or hyperbolic

steady states   diffusion processes   wave phenomena

Unlike ODEs, PDEs can not be analyzed "uniformly" and there is no general PDE solver.
Numerical methods for PDEs is a very vast subject and a vast area of reasearch.
Given the limited amount of time, we look only at few examples...

Ex.: (1) Tubular reactor ⇝ Slides

(2) Steady state tubular reactor ⇝ Slides

(3) Time dependent tubular reactor

⇝ Slides (Exercises!)
on 21.11.23

Once more: this is a vast subject and we only glanced at the subject
Other methods: Finite Volume
Finite Element
Discontinuous Galerkin (DG)

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