IV. Partial Differential Equations

Practical problems often depend on more than one variable, e.g. time (t) and space (x, y, z). This leads to so-called Partial Differential Equations (PDEs).

Classical examples:

(i) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ Laplace eq.

(ii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y)$ Poisson eq.

(iii) $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ Heat eq. (Diffusion)

(iv) $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0$ Wave eq.

In order to form well-posed problems, PDEs have to be supplemented by appropriate boundary conditions and sometimes also with initial conditions.

The above (linear) PDEs can be classified as elliptic, parabolic or hyperbolic

elliptic / steady states; parabolic \ diffusion processes; hyperbolic \ wave phenomena