



MATLAB: $[Q, R] = \text{qr}(A)$

With this: Let $A = QR$

$$\|A\vec{x} - \vec{b}\|_2^2 = \|\vec{v}\|_2^2$$

(Q ortho!) $\|Q^T(A\vec{x} - \vec{b})\|_2^2 = \|Q^T\vec{v}\|_2^2 = \|\vec{v}\|_2^2$

$$\|Q^T(QR\vec{x} - \vec{b})\|_2^2 = \|\vec{v}\|_2^2$$

$$\|R\vec{x} - Q^T\vec{b}\|_2^2 = \|\vec{v}\|_2^2$$

$$\left\| \begin{pmatrix} R_1 \\ \dots \\ 0 \end{pmatrix} \vec{x} - \begin{pmatrix} \vec{c} \\ \vec{d} \end{pmatrix} \right\|_2^2 = \|\vec{v}\|_2^2$$

split

$$\|R_1\vec{x} - \vec{c}\|_2^2 + \|\vec{d}\|_2^2 = \|\vec{v}\|_2^2$$

n equations in n unknowns $\leadsto \vec{x} = R_1^{-1} \vec{c}$
invertible because $r_{ii} \neq 0$ ($i=1, \dots, n$)