In summary: ① $A = QR$, $Q^T\vec{b} = \begin{pmatrix} \vec{c} \\ \vec{d} \end{pmatrix}$

② $\vec{x} = R_1^{-1}\vec{c}$

③ $\|\vec{r}\|_2 = \|\vec{d}\|_2$

Ex.: (3) Same as ex. (2) but with QR

↝ slides

(Of course, we get the same result!)

(4) Same as ex. (2) & (3) but with the MATLAB all-in-one solution: $\vec{x} = A \backslash \vec{b}$

↑ backslash operator

V.2 Non-linear Least-Squares

Let's now consider the case of overdetermined nonlinear systems of equations

$$\begin{aligned} f_1(x_1,\dots,x_n) &= b_1 \\ f_2(x_1,\dots,x_n) &= b_2 \\ &\vdots \\ f_m(x_1,\dots,x_n) &= b_m \end{aligned}$$

or short $\vec{F}(\vec{x}) = \vec{b}$

$\vec{F}: D \subset \mathbb{R}^n \to \mathbb{R}^m$

$\vec{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix} \in \mathbb{R}^m$

$\vec{x} = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^n$

m (nonlinear) equations in n unknowns.

Usually $m \gg n$ (many more measurements than parameters!)