

Least-squares solution:

$$\min_{\vec{x} \in D} \phi(\vec{x})$$

where $\phi(\vec{x}) = \frac{1}{2} \|\vec{f}(\vec{x}) - \vec{b}\|_2^2$

let's rewrite $\phi(\vec{x})$:

$$\begin{aligned} \phi(\vec{x}) &= \frac{1}{2} \|\vec{f}(\vec{x}) - \vec{b}\|_2^2 \\ &= \frac{1}{2} \sum_{i=1}^m (f_i(\vec{x}) - b_i)^2 \end{aligned}$$

(not sufficient... see below)

A necessary condition for a minimum is that the gradient of ϕ vanishes:

$$\vec{\nabla} \phi = \begin{pmatrix} \frac{\partial \phi}{\partial x_1} \\ \vdots \\ \frac{\partial \phi}{\partial x_n} \end{pmatrix} \stackrel{!}{=} 0$$

$$\begin{aligned} \text{Then } \frac{\partial \phi}{\partial x_j} &= \frac{\partial}{\partial x_j} \left(\frac{1}{2} \sum_{i=1}^m (f_i(x_1, \dots, x_j, \dots, x_n) - b_i)^2 \right) \\ &= \sum_{i=1}^m (f_i(\vec{x}) - b_i) \cdot \frac{\partial f_i}{\partial x_j} \stackrel{!}{=} 0, \quad j=1, \dots, n \end{aligned}$$

These are n (nonlinear) equations for the n unknown parameters