

The vanishing gradient is only a necessary condition for a minimum (it could also be a maximum or a saddle point)

We need one further condition:

$$H = \underset{\text{Hessian matrix}}{\text{Hess } \phi} = \left(\frac{\partial^2 \phi}{\partial x_i \partial x_j} \right)_{i,j} \text{ positiv definit}$$

i.e. $\vec{x}^T H \vec{x} > 0$ for all $\vec{x} \in \mathbb{R}^n, \vec{x} \neq 0$.

In one variable, this reduces to the familiar

cases:

