V.2.1 Newton method

Idea: apply Newton's method directly to

$$\vec{\nabla}\phi \overset{!}{=} 0$$

Linearize $\vec{\nabla}\phi$ around some given $\overset{\overset{\text{k-th iteration}}{}}{\vec{x}^{(k)}}$:

$$\vec{\nabla}\phi(\vec{x}^{(k)}) \approx \vec{\nabla}\phi(\vec{x}^{(k)}) + (\text{Hess}\,\phi)(\vec{x}^{(k)})(\vec{x} - \vec{x}^{(k)})$$

Define $\overset{\text{(k+1)-th iteration}}{\vec{x}^{(k+1)}}$ as

$$\vec{\nabla}\phi(\vec{x}^{(k)}) + (\text{Hess}\,\phi)(\vec{x}^{(k)})\underbrace{\left(\vec{x}^{(k+1)} - \vec{x}^{(k)}\right)}_{\vec{s}\ \text{(call it } \vec{s}\text{!)}} = 0$$

Then

$$\vec{x}^{(k+1)} = \vec{x}^{(k)} + \vec{s}\ , \quad k = 0, 1, \ldots, \text{convergence}$$

$$\text{solve} \quad (\text{Hess}\,\phi)(\vec{x}^{(k)})\,\underline{\underline{\vec{s}}} = -\vec{\nabla}\phi(\vec{x}^{(k)})$$

Rem.: (i) if close enough to <u>a</u> minimum, it converges quadratically $p=2$

existence & uniqueness discussion in chapter II!

(ii) needs the Hessian matrix