

## V.2.2 Gauss-Newton method

Idea: linearize the residual equations  $\vec{F}(\vec{x}) - \vec{b}$  and solve a sequence of linear least-squares problems

So linearize at some given  $\vec{x}^{(k)}$ :

$$\vec{F}(\vec{x}) - \vec{b} \approx \underbrace{\vec{F}(\vec{x}^{(k)}) - \vec{b}}_{\vec{\beta}^{(k)}} + \underbrace{J(\vec{x}^{(k)})}_{A^{(k)}} (\vec{x} - \vec{x}^{(k)})$$

← k-th iteration

$$\leadsto A^{(k)} \vec{x} - \vec{\beta}^{(k)}$$

Simply rearranging the terms... and a linear least-squares problem appears!

So instead of solving the nonlinear least-squares problem, one solves a sequence of linear least-squares problems:

$$\begin{array}{l} \vec{x}^{(0)} \longrightarrow \min_{\vec{x} \in \mathcal{D}} \frac{1}{2} \| A^{(0)} \vec{x} - \vec{\beta}^{(0)} \|_2^2 \longrightarrow \vec{x}^{(1)} \\ \text{initial guess} \\ \min_{\vec{x} \in \mathcal{D}} \frac{1}{2} \| A^{(1)} \vec{x} - \vec{\beta}^{(1)} \|_2^2 \longrightarrow \vec{x}^{(2)} \\ \vdots \\ \min_{\vec{x} \in \mathcal{D}} \frac{1}{2} \| A^{(k)} \vec{x} - \vec{\beta}^{(k)} \|_2^2 \longrightarrow \vec{x}^{(k+1)} \end{array}$$

Stop when  $\| \vec{x}^{(k+1)} - \vec{x}^{(k)} \| < \text{tolerance}$