Instead of solving a LSEs, the IP can also be found directly by the Lagrange Interpolation formula (LI)

$$p_n(x) = \sum_{j=0}^{n} y_j \cdot L_j^n(x)$$

where

$$L_j^n(x) = \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i} \qquad \text{for } j = 0, 1, \dots, n$$

are the so-called Lagrange polynomials (LPs).

The LPs have the following properties:

(LP1) $L_j^n(x)$ is a polynom of degree n

(LP2) $L_j^n(x_k) = \delta_{jk} = \begin{cases} 1, & j = k \\ 0, & j \neq k \end{cases}$

(LP2) is the reason why the LI fulfills the ICs:

$$p_n(x_i) = \sum_{j=0}^{n} y_j \cdot L_j^n(x_i)$$

$$= 0 + \dots + 0 + y_i \cdot \underbrace{L_i^n(x_i)}_{1} + 0 + \dots + 0$$

$$= y_i \checkmark$$